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TEST EVALUATION REPORT

PROBABILISTIC MANPOWER PLANNING FOR THE RESEARCH AND DEVELOPMENT ORGANIZATION

by

Larry H. Johnson

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U.S. ARMY MISSILE COMMAND

Redstone Arsenal, Alabama

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Research and Technology Office
Test and Reliability Evaluation Laboratory
Research and Engineering Directorate (Provisional)
U. S. Army Missile Command
Redstone Arsenal, Alabama 35809

ABSTRACT

There is an existing need for development of techniques for solving inventory problems when the parameters can be specified only in a probabilistic sense. Such a technique was developed in this study which utilized both linear and dynamic programming to operate on the probabilistic data. The problem addressed was manpower for a research and development organization which bids competitively for research and development tasks; however, the technique developed is applicable as well to machines, materials, or any other type of inventory.

Probabilistic planning is visualized as a process for continually and systematically evaluating the manpower requirements and recognizing the risks being taken with any given manpower plan. This author's approach to the manpower problem provides:

1) An array of all possible workloads

2) The probability associated with each

3) The minimum cost approach to performing each workload

4) A corporate manpower plan which allows the corporation to adjust its manpower to the actual workload, when it occurs, with least regret.

Mathematical models developed are suitable for solution on electronic computers and provide management with a rapid evaluation of possible management decisions. The corporation is thus provided with a mathematical simulation of the manpower planning system which can be utilized to readily evaluate effects of various inputs and promote better understanding of the problem at hand.

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CHAPTER I

INTRODUCTION AND PURPOSE

Exploits of modern man in the world of business have stimulated a revolution in competitive strategies and management techniques in an effort to maximize capital gain and professional status of the corporation. As time progresses competition becomes more acute, and the need for advanced management tools becomes paramount. No firm remains unaffected by the forces of progress, and the security of the corporation depends a great deal upon its ability to predict the future and concentrate its efforts toward profitable goals. Management of a corporation which bids competitively for government research and development (R&D) contracts requires control systems and analytical planning tools which are seemingly unique and worthy of special consideration.

What is meant by "research and development"? The term cannot be adequately defined for universal application for it covers a wide range of activities. The Department of the Army defines it as an effort to develop weapons, equipment, and techniques which are qualitatively superior to those of any potential enemy, in any environment, and under all conditions of war, to enable American forces to carry out their national security missions with

maximum effectiveness.[1] Research and development may be subdivided into four categories between which there are no distinct dividing lines:

Basic Research — an investigation or study of basic scientific laws and phenomena when there is no preconceived value for their utility.

Supporting Research — a study of specific scientific laws and phenomena with preconceived notions regarding their utility at some future time.

Applied Research — a studious inquiry or investigation of scientific laws and phenomena for which there is a present practical objective.

Development — a systematic use of existing scientific knowledge directed toward the design or improvement of items to meet specific performance requirements.

This author's investigation was directed primarily toward corporations engaged in either applied research or development, although the planning techniques which evolved are also applicable to basic and supporting research activities.

Shannon[2] investigated a system for management control of R&D activities in which he indicated that R&D managers perform basically the same functions of planning, implementation, and evaluation as do managers of any other

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[1] Army Regulation Number 705, "Research and Development of Material," Headquarters, Department of the Army, Washington 25, D. C., 14 January, 1963.

[2] R. E. Shannon, "A System for the Control of Research and Development Activities," M. S. Thesis, University of Alabama, Tuscaloosa, Alabama, May, 1960, pp. 75-76.

activity. However, unique differences exist by nature of the R&D activity and the type of individuals making up the organization, namely, scientific and engineering personnel. Hill[3] also described the R&D management problem and outlined some suggestions for improving the control and operational cost estimating techniques for technological development programs. Both authors agree that planning is one of the most important attributes of effective management control and that planning within an R&D organization introduces management to many problems not encountered in the established manufacturing industry. A basic difficulty is that R&D problems are rarely well defined, and existing management controls do not sufficiently account for uncertainty, which is the hallmark of R&D programs.

Why should one plan? Many arguments have been presented in the literature and have usually resulted in some optimization process for the utilization of manpower, facilities, time, or accumulated wealth. All of these are important; however, the real reason for planning is to gain the most from available capabilities and to establish direction and focus for these efforts.

There are many types of planning utilized by management. For example, planning may be associated with the function being planned, it may be related to the time or timing of various events, or it may be associated with mathematical models which are designed to simulate corporate activities.

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[3] L. S. Hill, "Towards an Improved Basis of Estimating and Controlling R&D Tasks," The Journal of Industrial Engineering, Vol. XVIII, No. 8, August, 1967.

Does planning pay off? The Stanford Research Institute studied the question "why companies grow," and concluded that in the case of both high-growth and low-growth companies, those that support planning programs have shown a superior growth rate over companies which do not support such a program.[4] Therefore, long-range planning is a management function which helps to make the corporation a progressive and long-living institution.

For the purpose of this study, long-range planning is defined as a process for continually and systematically making management type decisions, recognizing the risks being taken, and measuring the results of these decisions against realizations through feedback. The question for long-range planning is not what should be done tomorrow, but what can be done today to cope best with the uncertain tomorrow. Long-range planning is a management process designed to indicate which risks should be taken. Successful long-range planning provides the capability to take greater risks and improve the management performance. To plan successfully, however, management must understand the risk being taken and choose rationally among the possible risk-taking courses of action rather than plunge into uncertainty on the basis of intuition, hearsay, or previous experience.

The type of long-range planning of interest is the probabilistic manpower requirement for an organization which bids competitively for government R&D contracts.

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[4] D. W. Ewing, Long-Range Planning for Management. Harper and Row Publishers, New York, New York, 1964, p. 62.

The problem of determining future R&D manpower requirements is one that challenges management continuously, for they cannot know which bids will materialize, precisely when successful contracts will be initiated, or the corporation's ability to meet all R&D milestones within the allocated period of time. Their problem is to determine the most probable manpower requirement versus time and to adjust the available work force in accordance with the actual manpower requirement as it becomes known. If existing full-time employment is not sufficient to meet the demand, management may choose to have these employees work overtime, hire additional employees, subcontract a portion of the work, or choose some reasonable mix of these solutions. A mathematical model which considers the cost of feasible solutions and minimizes the total contract cost is presented in Chapter III. This model is identified as the Minimum Cost Method.

Since the workload is known only in a probabilistic sense, any corporation which implements recommendations of the minimum cost method is taking a calculated risk expressed as a function of workload and manpower level. In this application, risk is the expected cost of adjusting the corporate manpower to whatever workload actually occurs. As the workload becomes known, an adjustment or transition may be made and an appropriate minimum cost schedule followed. A mathematical model which minimizes the total expected risk associated with a recommended manpower plan is developed in Chapter IV and is identified as the Minimum Risk Method.

Considerable literature relating to the development of project and program planning models, development of optimum scheduling techniques, etc.,

has been published by various authors; however, little has been published on methods for the consolidation of planning information to produce a total corporate plan. Planning models which provide this consolidation and utilize feedback and decision points are in need of development.[5] The primary objective of this author's research is to develop such a model.

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[5] A. W. Wortham, "Probabilistic Long Range Planning - Development of Statistical Tecnniques for Forecasting Budgetary Requirements for Financing, Manpower and Facilities," The Journal of Industrial Engineering, Vol. XVII, No. 11, November, 1966, p. 556.

CHAPTER II

PROBABILISTIC MANPOWER REQUIREMENT

A. Statement of the Problem

If a corporation which bids on government R&D contracts is to fulfill its current contractual obligations and to perform the tasks required by contracts it will receive in the future, what will be the manpower requirement versus time? If it is assumed that the corporation will receive every contract for which it bids, the manpower requirement can readily be assessed. However, in most cases it cannot be assumed that the corporation will be awarded every contract, nor is it known for certain when the successful contracts will be initiated. If a subjective or conditional probability can be associated with these unknowns, the probability of requiring various levels of manpower can be calculated as shown in the three mathematical models developed in this chapter.

In manpower planning for an R&D organization, the manager must appreciate that mathematical models are merely an aid in the planning and selection process. Model making is an essential part of the process for understanding the situation at a given time, the elements of the situation being taken into consideration, and the concepts being used. Models are important in controlling a situation by suggesting system response to future environmental changes and management decisions. Models also allow the interjection of that important R&D element, uncertainty.

Uncertainty must usually be expressed as subjective judgement by the planner. His view of the future may take one of the four basic forms. [1]

Ignorance — He may see the future as a complete blank, finding himself unwilling or unable to make useful statements about it. Decisions made under such conditions have been described as "heroic" rather than "rational."

Assumed certainty — He pretends, for all practical purposes, that the future is exactly known. When he assumes certainty about the future, single-valued estimates which are called deterministic are used.

Probability — He may admit that he is not able to say exactly what is going to happen in the future, but he is able to say that one of several possible futures will occur with stated probabilities. The classic example is that of flipping a coin.

Intuition — His view of the future may suggest that a variety of events is possible, but he is unable to make any statements about their probability.

All four of these elements of uncertainty are important in R&D manpower planning; however, uncertainties which may be treated in a probabilistic manner are of primary interest in this thesis.

B. Assumptions

For application of the mathematical expressions developed in this thesis to manpower planning under uncertainty, the following assumptions must hold:

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[1] H. T. Darracott, "Report on Technological Forecasting," Defense Documentation Center Report Number AD 664165, June, 1967, p. 6-4.

1. A manpower array which indicates the total, time-adjusted manpower requirements can be generated. That is, for any project which is proposed or in progress, the required number of men can be defined by job classification and by time interval for the duration of the project. An example is shown in the manpower schedule in Table 1.

Table 1. Manpower Schedule for Project A

Quarter	1	2	3	4
Managers	2	2	1	1
Engineers	10	10	5	5
Technicians	20	20	30	30
Total	32	32	36	36

2. Each project is statistically independent of all other projects. For example, the award of a given R&D contract is not affected by the gain or loss of any other contract being considered.

3. Required personnel of common disciplines are reasonably interchangeable.

4. The subjective probability of receiving each outstanding bid can be assessed.

5. Each contract being considered has a proposed starting date. If this date is uncertain, the conditional probability of starting on various dates can be estimated.

Necessity for the preceding assumptions or conditions will become apparent as the mathematical planning models are developed in section D of this chapter.

C. Data Inputs

1. Development Schedules and Associated Manpower Requirements. In order to develop a time-adjusted manpower array, a program schedule must be generated for each project, and manpower requirements for accomplishing each phase of the project must be determined.

Two of the most commonly used techniques for presenting program schedules are Gantt[2] charts and PERT[3] diagrams. In both cases the program is characterized as a network of interrelated events which must be accomplished in sequence. For example, the Gantt chart requires first the division of the plan into a comprehensive set of operations which are interdependent to the extent that each must be completed before the project is completed. Each operation may be divided into a sequence of phases or events and their required order of accomplishment. Finally, the time required to perform each event in the project must be estimated and the total project plotted along an axis of calendar time. An example Gantt chart is presented in Figure 1. In PERT, the project is presented as a network of events and activities required to accomplish the end objective, T_o, as illustrated in Figure 2. In this flow diagram, events are depicted by circled numbers and activities by arrows. These

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[2] C. D. Flagle, "Probability Based Tolerances in Forecasting and Planning," The Journal of Industrial Engineering, Vol. XII, No. 2, March-April, 1961.

[3] D. G. Malcolm, J. H. Roseboom, C. E. Clark, and W. Fazar, "Application of a Technique for Research and Development Program Evaluation," Operations Research, Vol. 7, 1959.

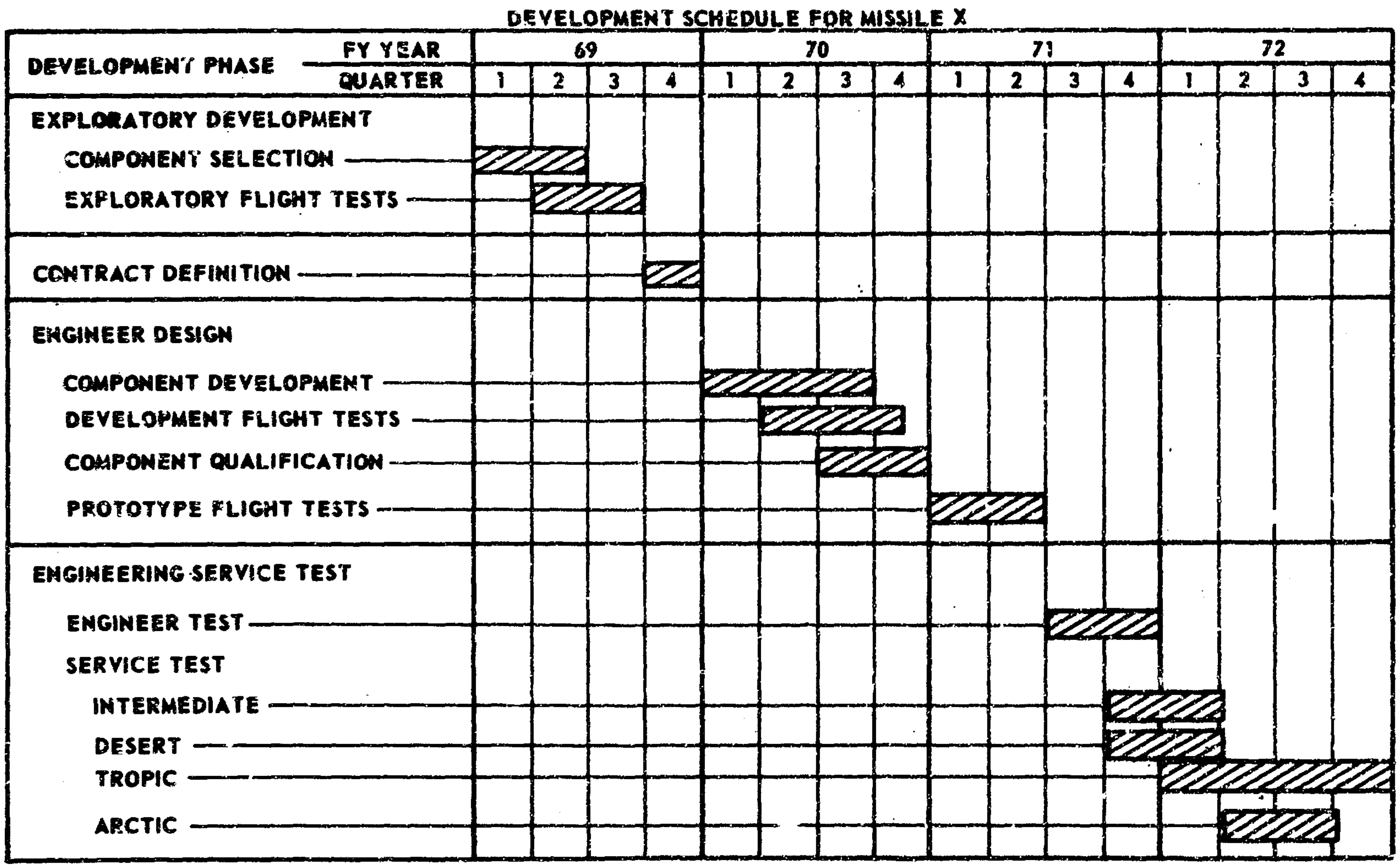

Figure 1. Gantt Chart Used for Presenting Schedule of Research and Development Programs

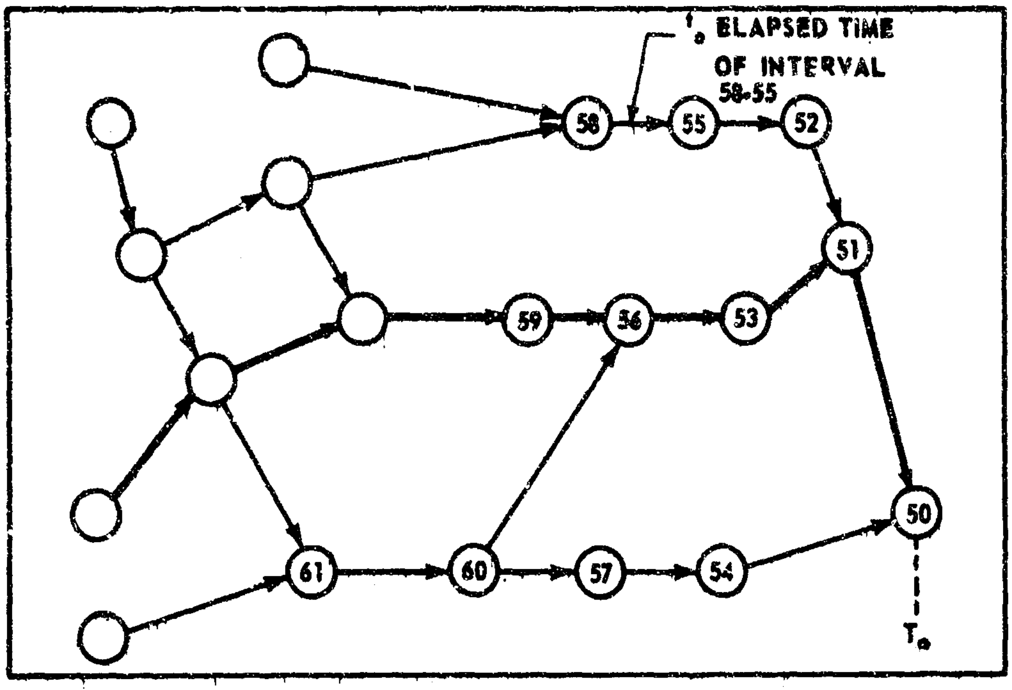

Figure 2. PERT System Flow Plan

activities must be performed in proper order as indicated by the network, and the expected time, t_e, required to conduct each activity must be estimated.

Both the Gantt and PERT techniques allow the planner to assess the probability of meeting given milestones on various dates; however, the probability associated with meeting program milestones is not addressed in this thesis. If the reader has interest in this area, it is suggested that he refer to the referenced literature. In general, the major program milestones are scheduled in R&D contracts with respect to contract initiation, and contract modifications are required if progress does not conform with the schedule.

A suitable coalition of the Gantt and PERT techniques is commonly used in scheduling R&D programs. A Gantt chart similar to Figure 1 is normally used by program management to monitor progress of the contract while planning groups utilize PERT to determine the most efficient sequence of activities for

meeting the scheduled milestones.

To determine the detailed manpower requirements for the contract, management must exercise judgement regarding the essential manning required for each network activity. An analytical technique developed by McGee and Markarian[4] allows the planner to allocate personnel by skill within preassigned manpower constraints in such a way that:

1. Minimum essential manning by skill required to carry out each activity in the PERT network is determined for the most austere conditions.

2. Maximum productive manning, by skill, which may be used most effectively to carry out the activities under a "crash" program is determined.

3. The most efficient allocation of manpower required to meet contractual milestones is determined.

The third property of the McGee and Markarian technique is the one most applicable to this thesis material. For example, if the heavy lines in Figure 2 represent the critical path for the PERT network and the McGee and Markarian technique is applied, the resultant is a time-adjusted manpower array as illustrated in Figure 3. This, of course, is the desired data input for probabilistic manpower planning.

2. Probability of Contract Award. The ideal response pattern for an organization is to receive every contract for which it bids. However, since this

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[4] A. A. McGee and M. D. Markarian, "Optimum Allocation of Research/Engineering Manpower Within a Multi-Project Organizational Structure," IRE Transactions on Engineering Management, September, 1962.

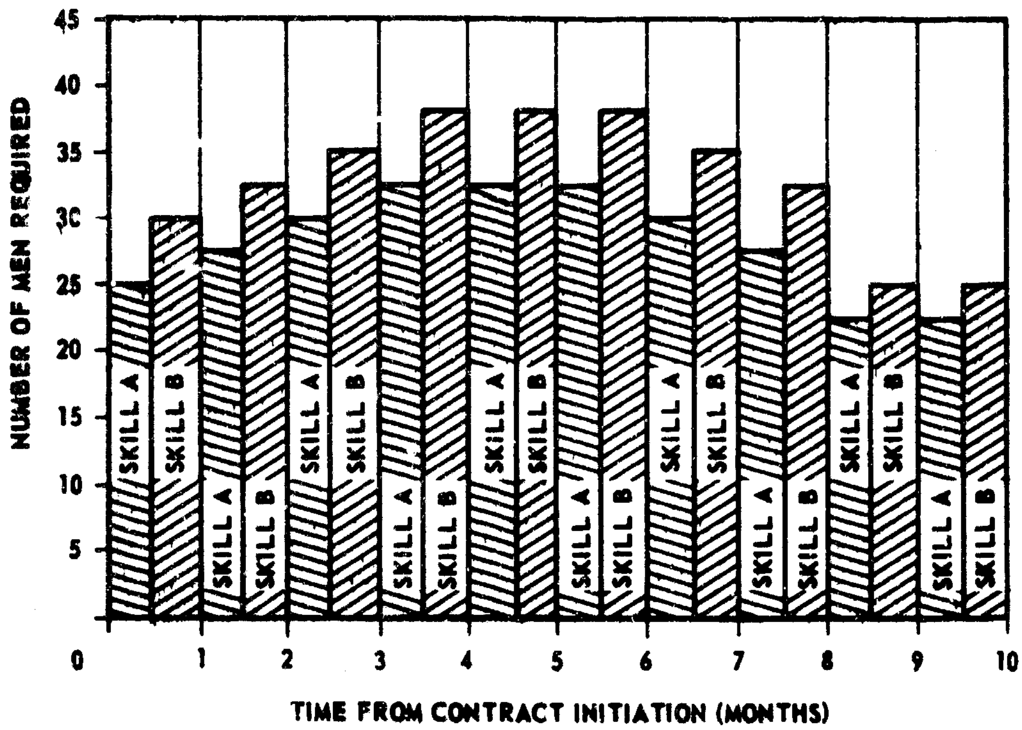

Figure 3. Time Adjusted Manpower Array for Project X

goal will probably never be attained, management requires some method for estimating the "capture probability" for each of the contract proposals. Techniques have been developed by Beckwith,[5] Dean,[6] and others for estimating this probability based on assumptions that men experienced in the state of the art of proposal preparation are available and that they can assess certain critical factors relevant to the job being considered such as competitive bidding corporations, contract awarding agencies, and the political or social

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[5] R. E. Beckwith, "A Stochastic Regression Model for the Proposal Success Evaluation," IEEE Transactions on Engineering Management, Vol. EM-12, No. 2, June, 1965, p. 60.

[6] B. V. Dean, "Contract Award and Bidding Strategies," IEEE Transactions on Engineering Management, Vol. EM-12, No. 2, June, 1965.

scene. A compilation of influential factors for the formal contract award process usually includes at least the following:

1. Technical capability (perhaps two or three components may be required, or a weighted average of several components)

2. Organizational structure or management quality for the task proposed

3. Performance record on past jobs for the agency releasing the request for proposal (RFP)

4. Experience in carrying out similar jobs

5. Geographical factor (e.g., "Does DOD currently favor the Boston area?")

6. Political factor (e.g., "How long has it been since this area has been awarded a major DOD contract?" or "Does this organization have an influential contact within DOD?")

7. Proposal quality relative to that expected from the competition

8. Price or profit margin

Management may or may not know the identity of competitive bidders. Models have been developed for both situations; however, the probability of contract award can be more accurately determined if the competitors are known. Bidding models are beyond the scope of this thesis and, consequently, are not discussed in detail. However, it is important that management becomes aware of these models and utilizes its contract-award synthesizing process at least to the point of statistical equivalence when dealing with probabilistic manpower planning.

Contrary to the formal contract award process noted above, Edward B. Roberts of Massachusetts Institute of Technology conducted a study[7] which indicated major differences between the formal process and the actual process for awarding government R&D contracts. Roberts indicated that the actual process is less competitive than stipulated by government regulations, and, with increased understanding of the actual process, a corporation can predict the probability of contract award with relatively high confidence.

For effective use of bidding models, management must be prepared to establish and maintain an organization for producing, accumulating, and processing RFP data on competitors for all types of R&D contract proposals. Without this commitment the process of estimating probability of contract award can at best be judgement based on the historical fraction of awards. The historical fraction may not be a valid estimate of the probability of award because of extenuating circumstances such as the eight factors previously noted.

Management should utilize the best means at its disposal for determining the probability of contract award. Manpower planning models developed in this chapter are sensitive to the probability of contract award, and erroneous judgement on the part of management can seriously affect the utility of the models.

3. Probability of Contract Initiation Date. If one assumes with certainty

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[7] E. B. Roberts, "How the U. S. Buys Research," International Science and Technology, September, 1964.

that a given R&D contract will be awarded, when will the contract be initiated? Exactly when the contract will be initiated cannot be determined usually because of contract negotiations, availability of funds, etc.; therefore, the concept of probability is again beneficial. If a conditional probability function can be associated with the contract initiation date, the probabilistic manpower planning model can be refined, as indicated in the third planning model developed in this chapter, and the future manpower requirement can be more accurately phased with time.

Unfortunately, scientific methods for estimating the probability function for date of contract initiation have not been developed. Consequently, management must depend on judgement and experience for this determination. Unlike the probability of contract award, however, errors in estimating the probability of contract initiation dates do not seriously affect the total manpower requirement. Instead, time phasing of the required manpower is erroneously predicted.

D. Development of Solutions

Probabilistic planning is a relatively new approach for estimating manpower requirements in industrial and governmental organizations. The procedures are statistical in nature and capitalize on the theory of expected values. For example, if X_t denotes a discrete random variable which can assume values of $X_{1,t}$, $X_{2,t}$, $X_{3,t}$ ... $X_{k,t}$ with respective probabilities p_1, p_2, p_3 ... p_k (where $p_1 + p_2 + p_3 + \ldots + p_k = 1$), the mathematical

expectation of X_t, denoted by $E(X_t)$ is defined as[8]

$$E(X_t) = p_1 X_{1,t} + p_2 X_{2,t} + p_3 X_{3,t} + \dots + p_k X_{k,t} \tag{1}$$
$$= \sum_{j=1}^{k} p_j X_j ,$$

where t is the planning period. For example, suppose a corporation has four outstanding proposals, each with a constant 25-percent chance of success. The required number of men per month per contract is 50, 40, 30, and 20 respectively. The expected manpower requirement is

(0.25) (50) + (0.25) (40) + (0.25) (30) + (0.25) (20) = 35 men per month.

It can readily be shown that planning for 35 men will not satisfy the needs of this corporation since the real possible outcomes are 0, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, and 140 men per month.

The significance of the above solution may best be explained by referring to another example, gambling. In gambling games the expected value of a win is the "fair" amount to pay for playing the game. In this sense, if one plays the game a large number of times, paying the fair amount each time, the amounts he wins and loses will average out.

It should be noted that the expected value is equivalent to the weighted arithmetic mean and may be equal to the most probable value only when the input data are selected from certain types of distributions.[9] The normal or

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[8] M. R. Spiegel, Schaum's Outline of Theory and Problems of Statistics. Schaum Publishing Company, New York, New York, 1961, p. 102.

[9] W. J. Dixon and F. J. Massey, Jr., Introduction to Statistical Analysis. McGraw-Hill Book Company, Inc., New York, New York, 1957, p. 355.

Gaussian distribution is an example where the expected, the most probable, and the 50-percentile values are all equal.

Since the expected manpower requirement does not represent a unique solution to the planning dilemma, as illustrated in the previous problem by the requirement for 35 men per month, what utility does it have? To evaluate the probable manpower requirement it is necessary to have some measure of the probable spread of results about the expected value. There are several such measures, but statistical variance $\left(\sigma_t^2\right)$ is preferred by most planning organizations.[10] Variance is usually converted to standard deviation $\left(\sigma_t\right)$ for meaningful expression.

How is the sample variance determined? By definition the variance $\left(\sigma_t^2\right)$ of a random variable $\left(X_t\right)$ is the expected value of the square of the difference between X_t and $E\left(X_t\right)$,[11] or symbolically

$$\sigma_t^2 = E\left\{\left[X_t - E\left(X_t\right)\right]^2\right\} \tag{2}$$

$$= E\left(X_t^2\right) - \left[E\left(X_t\right)\right]^2 . \tag{3}$$

Therefore, in the previous example problem where $E\left(X_t\right)$ was 35 men per month, the variance would be

$$\sigma_t^2 = \left[(0.25)(50)^2 + (0.25)(40)^2 + (0.25)(30)^2 + (0.25)(20)^2\right] - \left[(35)^2\right]$$

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[10] K. S. Packard, "Probabilistic Forecasting of Manpower Requirements," IRE Transactions on Engineering Management, Volume EM-9, September, 1962, p. 137.

[11] P. M. Morse and G. E. Kimball, Methods of Operations Research, The M. I. T. Press, Cambridge, Massachusetts, 1962, p. 21.

$= 1350-1225$

$= 125.$

The standard deviation for this example is $\sqrt{125}$, or 11 men per month. Since this standard deviation is fairly large compared with the expected value, approximately $1/3$, management is alerted that planning for a requirement of 35 men per month is a high risk situation. As may be suspected, relatively small standard deviations are desired.

At the beginning of this chapter a major planning problem was addressed, i.e., how may future manpower requirements be predicted in the light of uncertainty? Scientific methods for making mathematical estimates of the future manpower requirements have been developed and are presented in the following paragraphs.

1. Model I -- Defined Scope of Work. Organizations with rigidly controlled manpower requirements are not uncommon in American industry. For this reason, the first model presented is one which assumes unit probability for requiring exactly ${}_jM_i(t)$ men of type j for project i during the period t. Thus the manpower matrix is defined as

$$\left[{}_jM_i(t) \right] .$$

Manpower requirements can be assessed from this model by summing the appropriate columns and rows. For example, consider the manpower matrix shown in Table 2.

In this example ${}_.M_.(\text{Jan})$ is the total manpower requirement for the month of January, ${}_AM_.(\text{Jan})$ is the total manpower of type A required during

Table 2. Example Manpower Matrix for Model I

Project (i)	Time Period (t) January Manpower Type (j) A	B	C	February Manpower Type (j) A	B	C	Σ
I	5	10	10	20	30	40	115
II	10	15	30	10	15	30	110
III	25	35	50	35	35	35	215
Σ	40	60	90	65	80	105	440

$._{\cdot}M.$ (Jan) = 40 + 60 + 90 = 190

$_{A}M.$ (Jan) = 40

$_{A}M_{I}$ (Feb) = 20

$.M.$ (.) = 440

the month of January, $_{A}M_{I}$ (feb) is the manpower of type A required for project I during the month of February, $.M.$ (.) is the total manpower requirement for the corporation, etc. As previously indicated, any desired estimate of the manpower requirements can be determined by addition of the proper row or column entries.

At a glance, Model I seems very basic and appears to be of little value; however, it is prob[illegible] the one most commonly used in industry today. Managers frequently decide which projects to include in the corporate plan and which to exclude, thus forcing this particular model.[12] Model I will also be

[12] A. W. Wortham, "Probabilistic Long Range Planning - Development of Statistical Techniques for Forecasting Budgetary Requirements for Financing, Manpower and Facilities," Journal of Industrial Engineering, Vol. XVII, No. 11, November, 1966, p. 554.

used as a building block for developing probabilistic planning models.

2. Model II – Probabilistic Scope/Inflexible Time Scale. The second model to be considered is basically an extension of Model I. It associates a subjective capture probability with each outstanding contract proposal; i.e., for every project i there is a probability p_i that the corporation will receive the contract. The expected manpower requirement is then given by

$$E\left(X_t\right) = \sum_{i=1}^{n} p_i \, {}_jM_i(t), \quad j = 1, 2, 3 \ldots m \tag{4}$$

where

p_i is the probability of capture for the i^{th} contract proposal.

j is the type of manpower or the various skills required.

i is the identification of the contract proposal.

t is the time period of interest.

${}_jM_i(t)$ is the number of men of type j required for project i during the period t.

n is the number of outstanding contract proposals.

The expected manpower matrix for Model II is identical to that for Model I except that each row is multiplied by the probability of contract award. For example, if the three projects presented in Table 2 have estimated probabilities of acceptance of 0.3, 0.4, and 0.5, respectively, the expected manpower is determined to be as shown in Table 3. Note that the manpower requirements have been reduced considerably from those presented in Table 2.

Table 3. Example Manpower Matrix for Model II

Project (i)	Time Period (t)						
	January			February			
	Manpower Type (j)			Manpower Type (j)			
	A	B	C	A	B	C	Σ
I	1.5	3.0	3.0	6.0	9.0	12.0	34.5
II	4.0	6.0	12.0	4.0	6.0	12.0	44.0
III	12.5	17.5	25.0	17.5	17.5	17.5	107.5
Σ	18.0	26.5	40.0	27.5	32.5	41.5	186.0

Given: $p_I = 0.3$, $p_{II} = 0.4$, $p_{III} = 0.5$

$_{\cdot}M_{\cdot}(\text{Jan}) = 84.5$

$_{A}M_{\cdot}(\text{Jan}) = 18$

$_{A}M_{I}(\text{Feb}) = 6$

$_{\cdot}M_{\cdot}(\cdot) = 186$

As previously indicated, the expected manpower requirement given by equation (4) does not necessarily provide management with all the information required for reliable planning procedures. Some measure of confidence for the expected manpower prediction is required. If there are a large number of outstanding proposals, it may be assumed that the probabilistic manpower requirement is normally distributed: however, if the number of outstanding contract proposals is small, the exact sample distribution must be examined to assess the probabilistic spread of the manpower requirement.

If the number of outstanding proposals is large, the variance of the manpower requirement about the expected value may be derived from equation (3). The variance for Model II is

$$\sigma_t^2 = \sum_{i=1}^{n} p_{ij} M_i^2(t) - \left[\sum_{i=1}^{n} p_{ij} M_i(t) \right]^2 \qquad (5)$$

To illustrate, the variance for the manpower of type A (Tables 2 and 3) during the month of January may be calculated as

$$\sigma^2 = \left[(0.3)(5)^2 + (0.4)(10)^2 + (0.5)(25)^2\right] - \left[(18.0)^2\right]$$

$$\sigma^2 = 345 - 324$$

$$\sigma^2 = 21\ ,$$

and the standard deviation is $\sqrt{21}$, or 4.6 men. Management may then choose to express the type A probabilistic manpower requirement for January as 18 ±5 men, or the manpower requirement may be expressed as a probability density function (Figure 4). The probability density function is calculated from the expected manpower requirement (μ) and the standard deviation (σ) by assuming that the probabilistic manpower requirement is normally distributed such that[13]

$$p\left(X_t\right) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{X_t - \mu}{\sigma}\right)^2}\ , \qquad (6)$$

where $\pi = 3.14159$, $e = 2.71828$, and unit area exists under the curve.

Because the average business manager is not well versed in statistics, the following explanation of the probabilistic manpower requirement is presented.

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[13] P. G. Hoel, Introduction to Mathematical Statistics, John Wiley and Sons, Inc., New York, New York, 1963, p. 101.

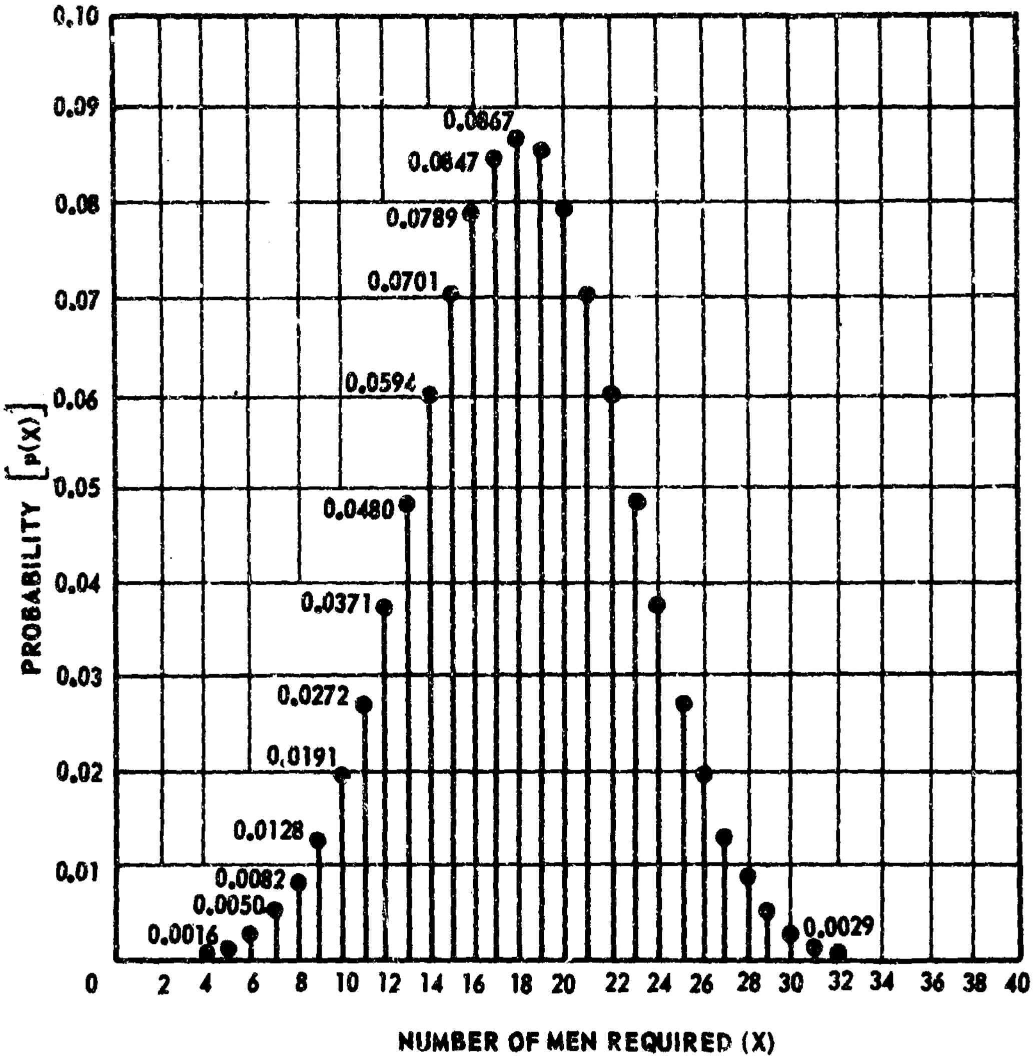

Figure 4. Probabilistic Manpower Requirement: Type A, January

If management desires to know the probability of requiring exactly X men of type A in the month of January, the probability may be read directly from Figure 4. Data in this figure indicate that there is an 8.67-percent chance that exactly 18 men will be required and that the probability decreases as the manpower requirement increases or decreases from the expected value.

If a conservative estimate of the required manpower is desired, i.e., what is the confidence that X men will be sufficient to satisfy the contractual

manpower requirement, a cumulative probability distribution function must be utilized. As indicated in Figure 5, management may be approximately 75 percent confident that 21 men of type A will satisfy the January requirement, approximately 90 percent confident that 24 men will be sufficient, etc. Even though the actual contractual requirements for January may be 0, 5, 10, 15, 25, 30, 35, or 40 men of type A, management can be 94 percent sure of meeting the January requirement if 25 men are estimated, which is also the number of men required if contract proposal III is successful.

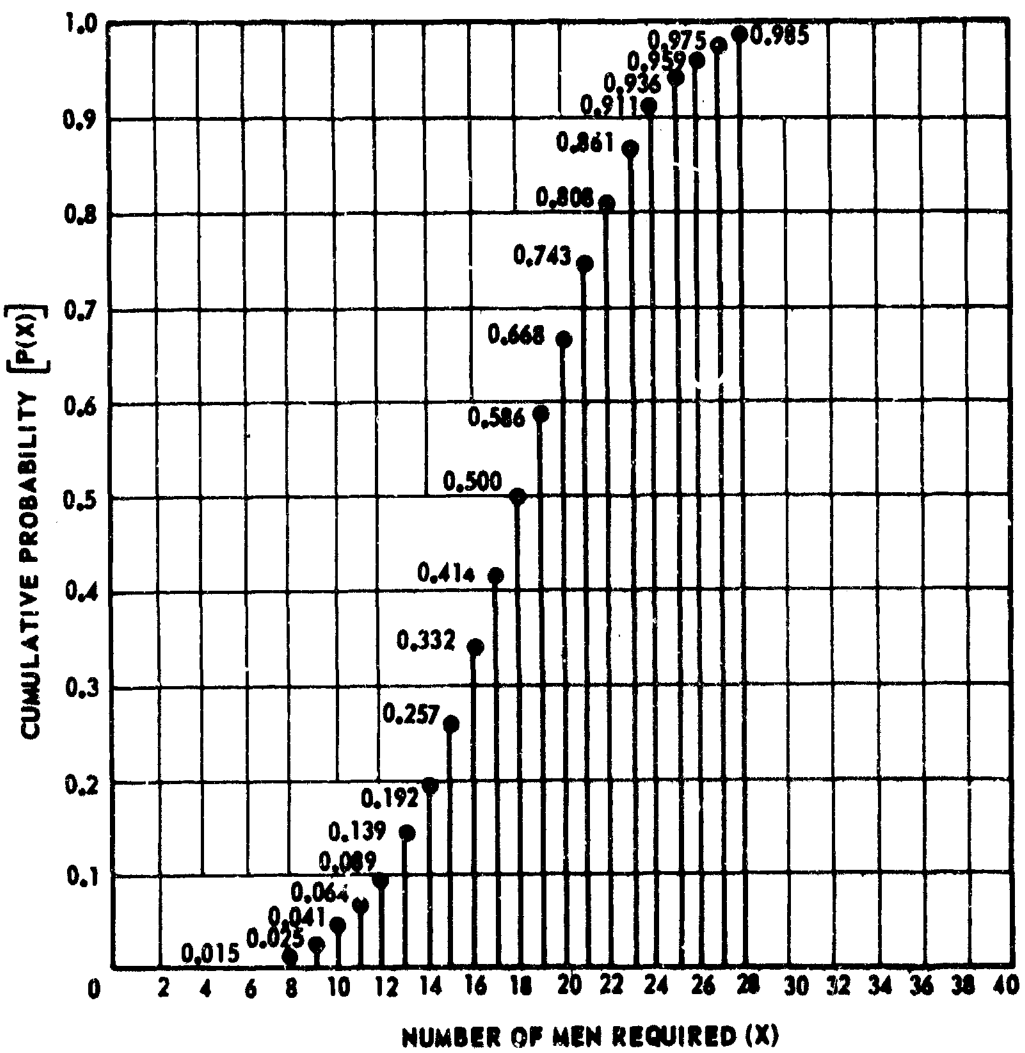

Figure 5. Cumulative Probabilistic Manpower Requirement: Type A, January

The cumulative probability distribution function is calculated by integration of equation (6) to determine the area under the curve as depicted in Figure 6. The cumulative probabilistic manpower requirement is therefore given by

$$P(X) = \int_{0}^{X} p(X)\, dX \tag{7}$$

or

$$P(X) = \frac{1}{\sigma\sqrt{2\pi}} \int_{0}^{X} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} dX \;. \tag{8}$$

The cumulative probabilistic manpower curve is one of the most powerful tools available for estimating the future manpower requirement.

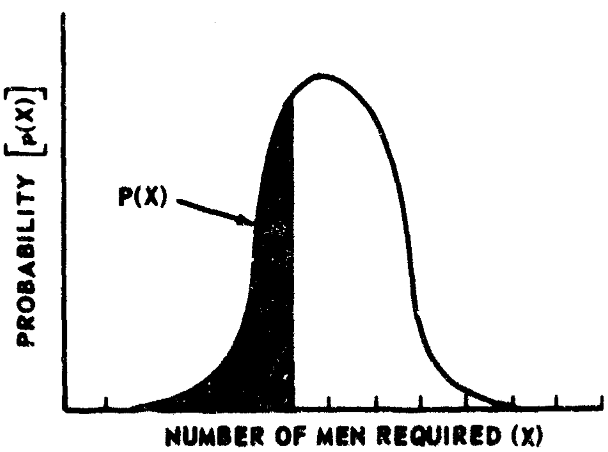

Figure 6. Graphical Presentation of Cumulative Probability

In general, a corporation which bids on R&D contracts does not have a large number of contract proposals outstanding at any given time. Therefore, it may be unreasonable to assume that the probabilistic manpower requirement

is normally distributed. This planning problem may be solved by the technique of complete enumeration,[14] which evaluates the probability associated with every possible combination of the manpower requirements. The expected number of men needed for any time period and a cumulative probability function may be determined by the enumeration method.

The complete enumeration method may best be explained by referring to an example problem. Consider the previous example (Tables 2 and 3) with three contracts and contract award probabilities as shown in Table 4.

Table 4. Outline of Manpower: Probability Matrix

Contract	January Manpower Type A	Probability
I	5	$p_1 = 0.3$
II	10	$p_2 = 0.4$
III	25	$p_3 = 0.5$

If contract I is acquired, 5 men of type A will be required during the month of January, and the probability of receiving contract I is 0.3. If contract I is not acquired, no (0) men of type A will be required, and the probability of not receiving contract I is $(1-p_1)$ or 0.7. If the award of contract I is indicated by M_1 and loss of contract I is indicated by M_1^*, and p_1 indicates the probability of receiving contract I and p_1^* indicates the probability of not receiving the job, the two possible outcomes for contract I are as shown in Figure 7.

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[14] J. F. Koonce, "Probabilistic Manpower Forecasting," M. S. Thesis, Texas A&M University, College Station, Texas, May, 1966, pp. 7-25.

Figure 7. Possible Outcomes of Contract I

If contract I is acquired there are two possible outcomes for contract II, or if I is not acquired there are two possible outcomes for contract II. If both I and II are acquired, 5 + 10 or 15 men will be needed. The probability of both contract proposals I and II being successful is $(p_1)(p_2) = (0.3)(0.4)$ or 0.12. The four possible outcomes for proposals I and II are shown in Figure 8.

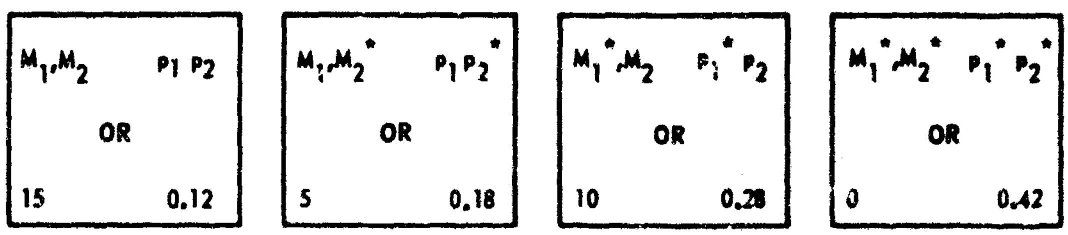

Figure 8. Possible Outcomes for Contracts I and II

Continuing the above process with contract III it is seen that eight combinations exist as shown in Figure 9.

The expected manpower requirement of type A during the month of January may be calculated from the results in Figure 9 and by equation (1) as follows:

$$E(M) = \sum_{s=1}^{8} p_s M_s$$

$$= (0.06)(40) + (0.06)(15) + \ldots + (0.14)(10) + (0.21)(0)$$

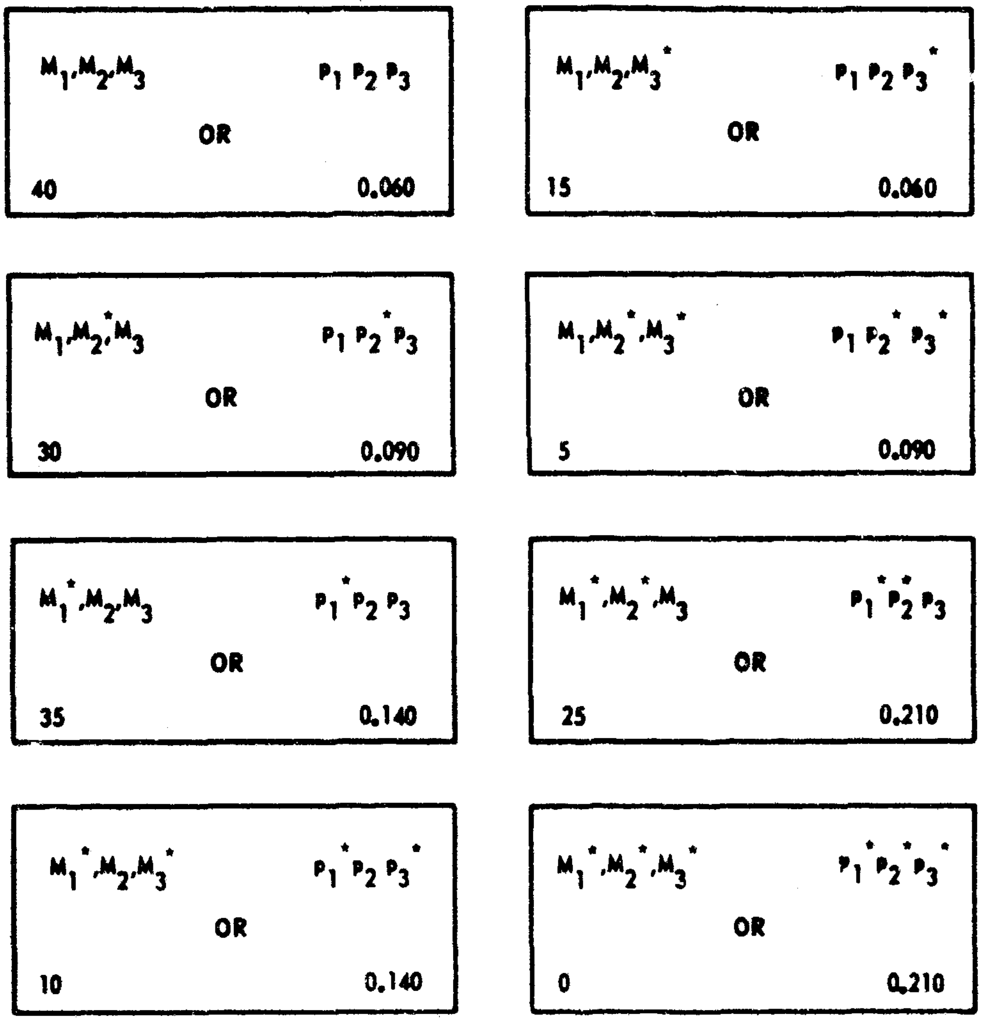

Figure 9. Possible Outcomes of Contracts I, II, and III

= 18 men.

A cumulative distribution for the probabilistic manpower requirement can also be determined from the results in Figure 9. The discrete cumulative distribution is depicted in Figure 10, which indicates that the probabilistic manpower requirements most likely are not normally distributed as assumed when this example problem was solved initially.

If the cumulative probabilities for intermediate manpower values are

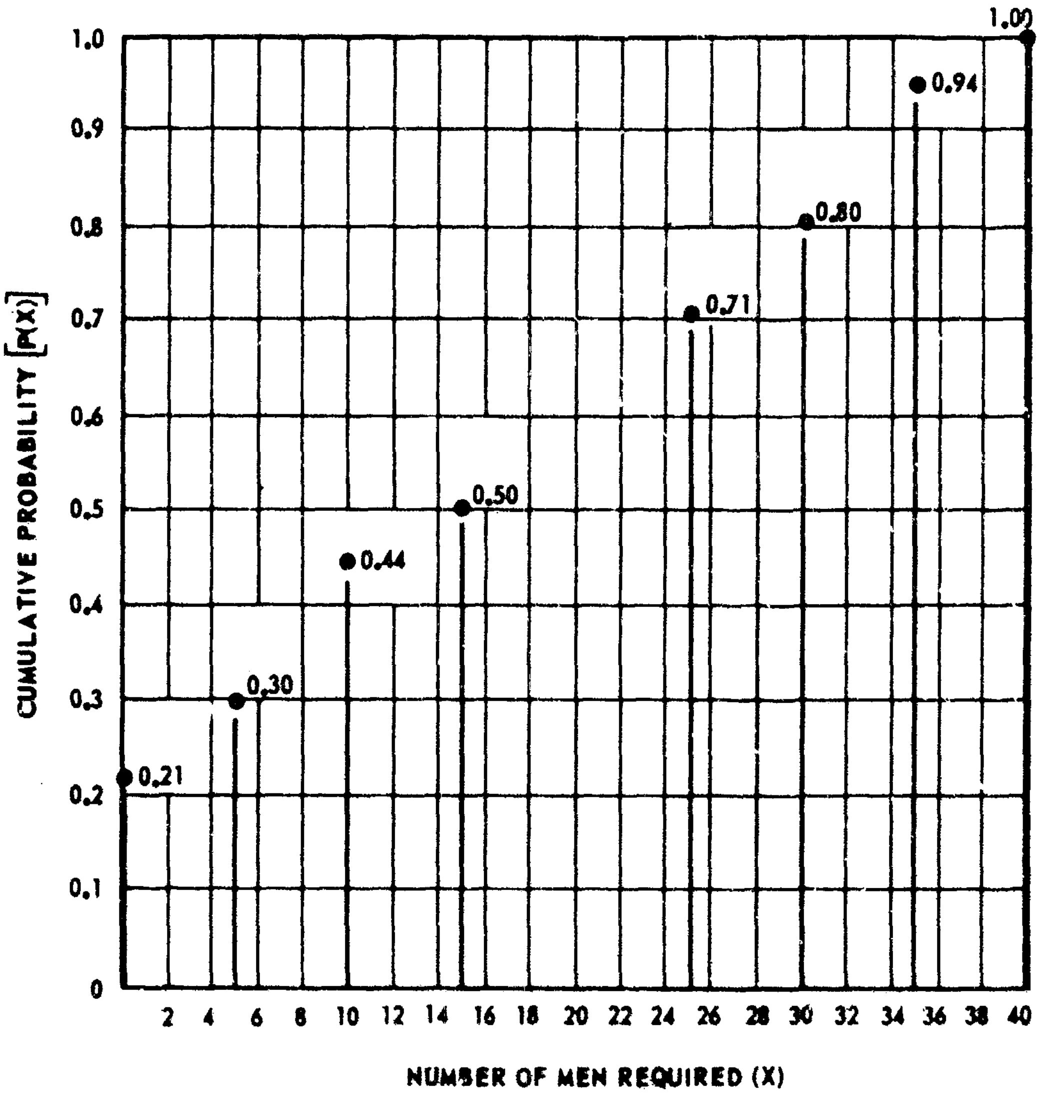

Figure 10. Cumulative Probabilistic Manpower Requirement: Type A, January

desired, they may be estimated by using straight-line interpolations between the known values. [5] Performing this operation on the eight data points in Figure 10 yields the cumulative distribution presented in Table 5.

For comparison of the enumeration method with the original method

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[5] J. F. Koonce, "Probabilistic Manpower Forecasting," M. S. Thesis, Texas A&M University, College Station, Texas, May, 1966, p. 8.

Table 5. Probabilistic Manpower Requirement: Type A, January

Number of Men (X)	Calculated Probability	Cumulative Probability
0	0.210	0.210
1		0.228
2		0.246
3		0.264
4		0.282
5	0.090	0.300
6		0.328
7		0.356
8		0.384
9		0.412
10	0.140	0.440
11		0.452
12		0.464
13		0.476
14		0.488
15	0.060	0.500
16		0.521
17		0.542
18		0.563
19		0.584
20		0.605
21		0.626
22		0.647
23		0.668
24		0.689
25	0.210	0.710
26		0.728
27		0.746
28		0.764
29		0.782
30	0.090	0.800
31		0.828
32		0.856
33		0.884
34		0.912
35	0.140	0.940

Table 5. Probabilistic Manpower Requirement: Type A, January (Concluded)

Number of Men (X)	Calculated Probability	Cumulative Probability
36		0.952
37		0.964
38		0.976
39		0.988
40	0.060	1.000

presented for Model II, it is interesting to note that the expected manpower requirement for both solutions is 18 men for the month of January; however, when the normal distribution is assumed for this small number of projects, the distribution about the expected value is misleading. For example, in the first solution it is estimated that management may be 94 percent confident of meeting the contractual requirements if 25 men are available, but when the actual distribution is evaluated it is estimated that management can only be 71 percent confident of meeting the contractual requirements with 25 men. Also, when the actual distribution is enumerated, it is determined that the expected value is not the most probable manpower requirement. The most probable manpower requirement is determined to be either 0 or 25 men rather than the expected 18 men. The significance of this result is that the original approach is valid only when there are a large number of contracts outstanding and when the assumption of normality is acceptable. Otherwise, the technique of complete enumeration should be employed.

In the enumeration process, it may be noted that there were two (2^1) possible outcomes for contract I, for contracts I and II there were four (2^2) possible outcomes, and for contracts I, II, and III there were eight (2^3)

possible outcomes. Consequently, it can be rationalized that for r contract proposals there are 2^r possible outcomes, and the probability associated with each possible outcome can be calculated.

As the number of outstanding contracts increases, the task of calculating by enumeration all of the possible outcomes and the associated probabilities for a given type of manpower becomes prohibitive. If, for example, there are ten outstanding proposals, there are 2^{10} or 1024 possible outcomes. If there is reason to suspect that the assumption of normal distribution is not acceptable, the enumeration technique should be utilized; however, to minimize the massive task of enumerating a large number of contracts, a variation of the complete enumeration technique is suggested. In the following paragraphs, methods for simplifying the enumeration process and estimating the expected manpower requirement and the cumulative distribution function are presented.

A binary numbering system may be used to simplify calculation of the 2^r possible outcomes and their corresponding probabilities. If 0 indicates that the contract will not be received, and 1 indicates award of the contract, binary tables such as those shown in Tables 6, 7, and 8 may be constructed. It is a relatively simple operation to continue adding projects in a systematic manner and construct a binary table of dimensions r by 2^r for any number of outstanding proposals.

Table 6. Binary Representation of One Outstanding Proposal

Project	Outcome Number	
	1	2
I	0	1

Table 7. Binary Representation of Two Outstanding Proposals

Project	Outcome Number			
	1	2	3	4
I	0	1	0	1
II	0	0	1	1

Table 8. Binary Representation of Three Outstanding Proposals

Project	Outcome Number							
	1	2	3	4	5	6	7	8
I	0	1	0	1	0	1	0	1
II	0	0	1	1	0	0	1	1
III	0	0	0	0	1	1	1	1

To calculate the manpower value $\left(M_s\right)$ for each tabulated outcome, the values of M_i are added for every case where a 1 exists in the column and the values of M_i* are added for every 0 in the column. For example, if the input data shown in Table 9 (which is the same problem presented in Table 4) are evaluated in accordance with Table 8, the required manpower for outcome number 1 is 0 men, for outcome number 2 is 5 men, etc., as outlined in Table 10.

Table 9. Outline of Manpower: Probability Matrix

Contract	${}_AM_i$ (Jan)	${}_AM_i^*$ (Jan)	p_i	p_i^*
I	5	0	0.3	0.7
II	10	0	0.4	0.6
III	25	0	0.5	0.5

Table 10. Determination of Manpower: Probability by Binary Technique

Project	Outcome Number (s)							
	1	2	3	4	5	6	7	8
I	0	1	0	1	0	1	0	1
II	0	0	1	1	0	0	1	1
III	0	0	0	0	1	1	1	1
${}_AM_s$ (Jan)	0	5	10	15	25	30	35	40
p_s	0.21	0.09	0.14	0.06	0.21	0.09	0.14	0.06

To calculate the probability $\left(p_s\right)$ associated with each tabulated outcome, the values of p_i* are multiplied for every 0 in the column and the values of p_i are multiplied for every 1 in the column. If the input data of Table 10 are evaluated in accordance with Table 8, the p_s for outcome number 1 is calculated by $p_1^* \cdot p_2^* \cdot p_3^*$, the p_s for outcome number 2 is calculated by $p_1 \cdot p_2^* \cdot p_3^*$, etc., until the p_s values for all tabulated outcomes are determined. The p_s values for this example problem are presented in Table 10. This method of calculating the probabilistic manpower requirements is much simpler and faster than the method illustrated in Figure 9.

If the number of outstanding proposals is large such that the cumulative distribution function can be closely approximated by the enumeration technique, the expected manpower requirement can be estimated by the area above the cumulative distribution function[16] as depicted in Figure 11. This approach is

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[16] P. M. Morse and G. E. Kimball, Methods of Operations Research. The M. I. T. Press, Cambridge, Massachusetts, 1962, p. 21.

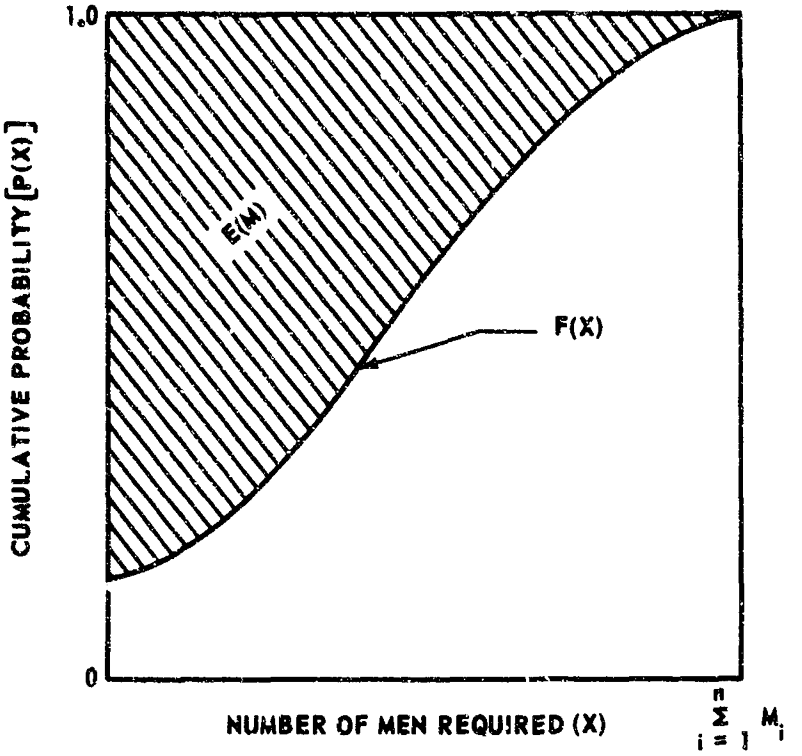

Figure 11. Expected Manpower Estimation

recommended since the usual method of computing the expected value by the equation

$$E(M) = \sum_{s=1}^{2^r} p_s M_s \tag{10}$$

becomes quite burdensome as the number of outstanding proposals (r) increases. The expected value may be calculated by one of two ways:

$$E(M) = \left(\sum_{i=1}^{n} M_i \right) - \left(\int_{0}^{\lambda} F(X)\, dX \right), \tag{11}$$

where

$$\lambda = \sum_{i=1}^{n} M_i ,$$

or

$$E(M) = \int_{y=F(0)}^{1} F(Y)\, dy , \qquad (12)$$

where

$Y = P(X)$.

$F(X)$ is the cumulative distribution function, in terms of X, fitted to the enumerated data points.

$F(Y)$ is the cumulative distribution function, in terms of $P(X)$, fitted to the enumerated data points.

$\sum_{i=1}^{n} M_i$ is the total number of men, of a given type, required during a specific time period if all of the n outstanding proposals are successful.

Equation (11) will be utilized to estimate the expected manpower requirement for the previous example problem for illustration. From the cumulative distribution data presented in Table 5, the expected manpower requirement is given by

$$\begin{aligned} E(X) = 40 - \Bigg[& \int_0^5 (0.018X + 0.210)\, dX + \int_5^{10} (0.028X + 0.160)\, dX \\ & + \int_{10}^{15} (0.012X + 0.320)\, dX + \int_{15}^{25} (0.021X + 0.185)\, dX \\ & + \int_{25}^{30} (0.018X + 0.260X)\, dX + \int_{30}^{35} (0.028X - 0.04)\, dX \\ & + \int_{35}^{40} (0.012X + 0.52)\, dX \Bigg] \end{aligned}$$

or

$$E(X) = 15.5 .$$

The estimate of 15.5 men differs somewhat from the expected value of 18 men calculated by complete enumeration. The estimated value is in error because the interpolating functions, $F(X)$ and $F(Y)$, are not exact representations of the actual cumulative function; however, as the number of contracts becomes large, the error in estimation may be accepted necessarily to avoid the task of calculation by complete enumeration.

With regard to the burdensome task of complete enumeration when there is a large number of outstanding contracts, a variation of the enumeration technique is suggested. A planning organization can usually determine the expected manpower requirements and the cumulative probability curves within acceptable tolerances by enumerating only a selected number of possible outcomes as required to approximate the cumulative distribution function and applying the above procedures. Techniques discussed in this section are adaptable to electronic data processing computers and consequently offer great utility to the progressive planning organization for estimating future manpower requirements.

3. Model III — Probabilistic Scope/Probabilistic Time Scale. The third planning model represents an extension of Model II and includes the subjective probability of contract initiation date.

A corporation which bids competitively on R&D contracts is not only uncertain of which contracts it will receive, but until a contract award is

actually made, the "go-ahead" date is also unknown. If a subjective probability can be associated with the expected award date for a contract, a more conservative estimate of the future manpower requirement can be realized.[17]

If p_i indicates the probability of receiving the i^{th} contract (as in Model II), and $p_{t/i}$ indicates the conditional probability that contract i will begin in time period t, the expected manpower equation for Model II may be amplified to

$$E(M) = \sum_{i=1}^{n} \sum_{t_o=0}^{v} p_i p_{t/i}\ {}_jM_i(t_o + k) , \tag{13}$$

where

$$\sum_{t_o=0}^{v} p_{t/i} = 1 \tag{14}$$

i is identification of the contract proposal.

n is the total number of outstanding proposals.

k is the time period after contract go-ahead $\left(k = t - t_o\right)$.

t_o is the period in which it is assumed that the contract go-ahead will be received.

t is the absolute time period $\left(t = t_o + k\right)$.

v is the total number of time periods over which the i^{th} contract may be initiated (the range of values for t_o).

${}_jM_i\left(t_o + k\right)$ is the number of men of type j required for project i during

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[17] L. B. Wadel and C. M. Bush, "An Approach to Probabilistic Forecasting of Engineering Manpower Requirements," IRE Transactions on Engineering Management, Vol. EM-8, No. 3, September, 1961.

the period t, assuming that the contract will be initiated in the t_o time period.

The best way to explain this model is to consider an example problem. Suppose it is known for certain $\left(p_i = 1\right)$ that a given contract will be received, that the required manpower of a given type will be as shown in Figure 12, and

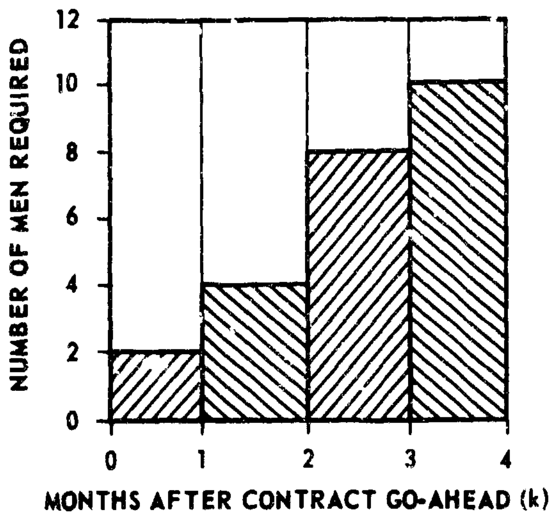

Figure 12. Manpower Requirements for Contract I

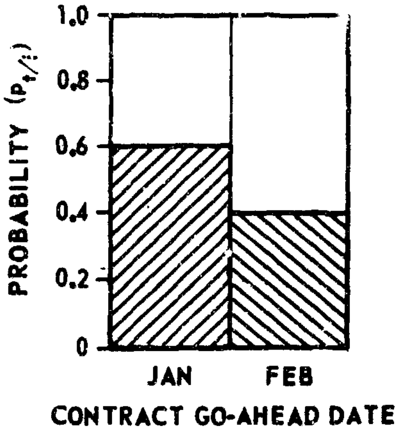

Figure 13. Probability of Go-Ahead Date: Contract I

the probabilities associated with expected contract initiation dates are 0.6 and 0.4 (Figure 13). If the contract is awarded in January, the manpower requirements will be 2, 4, 8, and 10 men per month (Figure 14). However since there is only a 60-percent chance that contract go-ahead will be received in January, the expected manpower requirements are 1.2, 2.4, 4.8, and 6 men per month (Figure 15). If the contract is not awarded in January but is awarded in February, the manpower requirements for the contract will be 0, 2, 4, 8, and 10 men per month (Figure 16). However, the expected manpower requirements will be 0, 0.8, 1.6, 3.2, and 4 men per month (Figure 17) due to

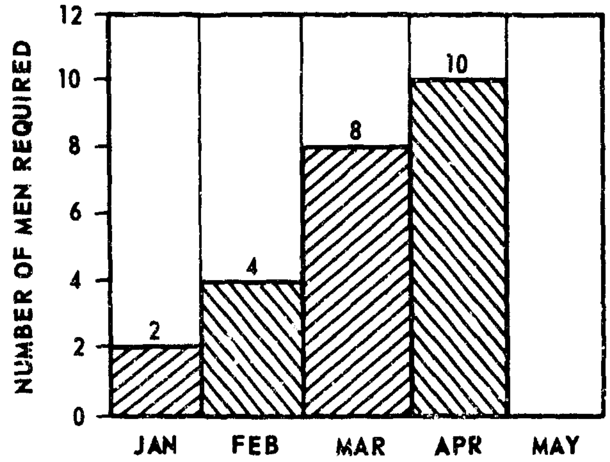

Figure 14. Contract I Manpower Requirements: January Go-Ahead

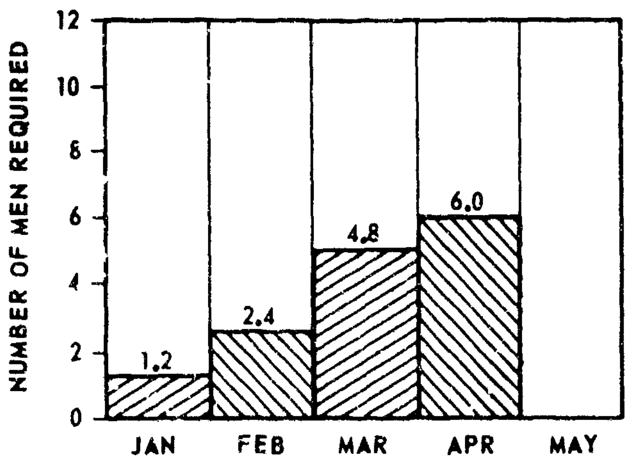

Figure 15. Contract I Expected Manpower Requirements: January Go-Ahead

the 40-percent chance of the contract being awarded in February. Since the total expected requirement for any month is the sum of the expected requirements for that month, the results in Figures 15 and 17 may be combined to develop the total expected manpower array shown in Figure 18. Review of the graphical presentation of this problem makes the mechanics for equation (13) easier to comprehend. For example, if equation (13) and the data given in Figures 12 and 13 are utilized to calculate the expected manpower requirement for the month of March, the model simplifies to

$$E\left(M_{March}\right) = \sum_{t_o = Jan}^{Feb} p_t \, M\left(t_o + k\right) \qquad (15)$$

$$= \left(p_{Jan}\right)\left(M_{2-3}\right) + \left(p_{Feb}\right)\left(M_{1-2}\right) .$$

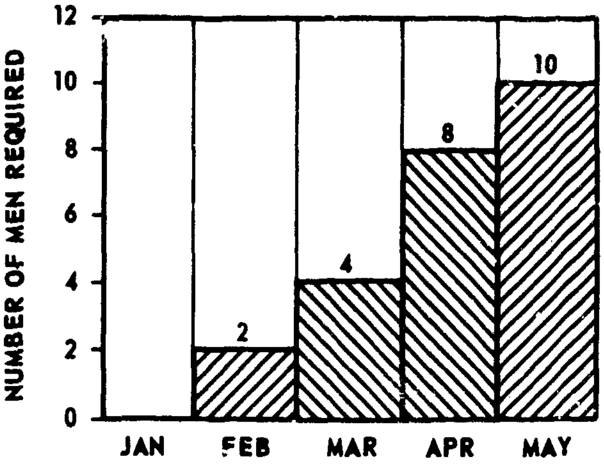

Figure 16. Contract I Manpower Requirements: February Go-Ahead

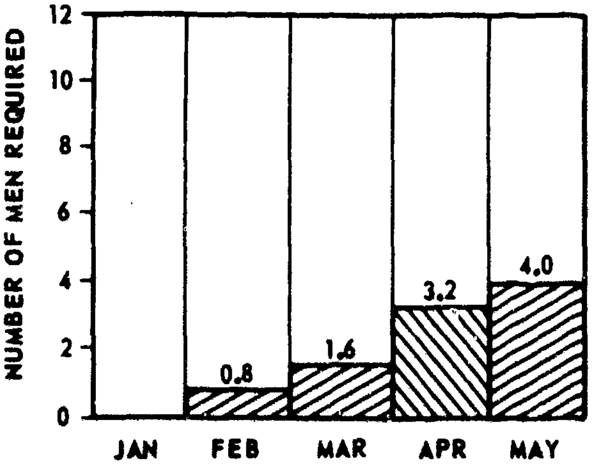

Figure 17. Contract I Expected Manpower Requirements: February Go-Ahead

Therefore, the solution is

$$E\left(M_{March}\right) = (0.6)(8) + (0.4)(4) = 6.4 \text{ men},$$

which is equivalent to the results presented in Figure 18. The mathematics may be streamlined by utilizing a $\left[M \times p_t\right]$ matrix similar to Table 11. The only input data required to develop such a matrix are the manpower requirements with respect to the contract go-ahead date (Figure 12) and the probability associated with each possible go-ahead date (Figure 13). The table entries are calculated as follows:

Step 1 — Multiply p_{Jan} by M_{0-1} and enter the result in the first column of the first row.

Step 2 — Multiply p_{Jan} by the succeeding M_k values and tabulate the results along the matrix diagonally (Table 11).

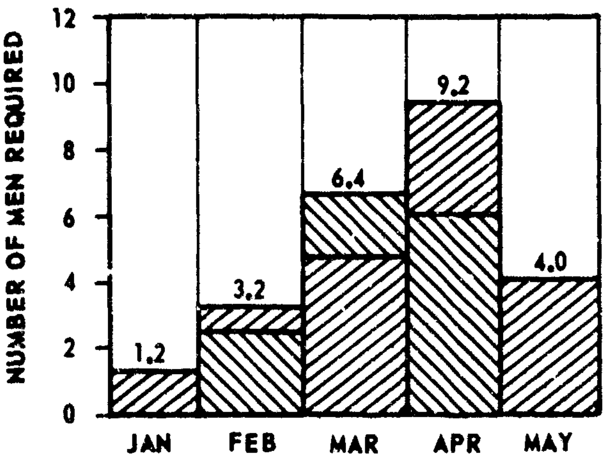

Figure 18. Total Expected Manpower Requirement for Contract I

Step 3 — Multiply p_{Feb} by M_{0-1} and enter the result in the second column of the first row.

Step 4 — Multiply p_{Feb} by the succeeding values of M_k and tabulate the results along the diagonal (Table 11). This process is continued until all v values of p_t are utilized and the expected manpower requirement for each month is determined by summing the respective columns within the table.

If a probability of capture had been associated with this example problem, the M column in Table 11 would have been modified by multiplying each M value by p_i and proceeding in the same manner demonstrated in the Appendix.

Table 11. Manpower Probability Matrix for Example Problem

k	M	Jan	Feb	Mar	Apr	May	June
	p_t	0.6	0.4	0	0	0	0
0-1	2	1.2	0.8				
1-2	4		2.4	1.6			
2-3	8			4.8	3.2		
3-4	10				6.0	4.0	
$E(M_t)$		1.2	3.2	6.4	9.2	4.0	0

If a large number of proposals are outstanding such that the normal distribution may be assumed for the probabilistic manpower requirement, the variance about the expected value may be determined by

$$\sigma^2 = \left[\sum_{i=1}^{n} \sum_{t_o=0}^{v} \left(p_i\, p_{t/i}\right) {}_jM_i^2 \left(t_o + k\right)\right] - \left[\sum_{i=1}^{n} \sum_{t_o=0}^{v} \left(p_i\, p_{t/i}\right) {}_jM_i \left(t_o + k\right)\right]^2 \qquad (16)$$

In the previous example problem, variance for the month of March about the expected value of 6.4 men is

$$\begin{aligned} \sigma^2 &= \left[(0.6)(8)^2 + (0.4)(4)^2\right] - \left[(0.6)(8) + (0.4)(4)\right]^2 \\ &= 44.8 - 41.06 \\ &= 3.74\ . \end{aligned}$$

The probabilistic manpower requirements may therefore be expressed as 6.4 $\pm\sqrt{3.74}$ (or 6.4 $\pm$1.93 men), and the cumulative probability function, assuming normality, is calculated by equation (8), where

$$P(X) = \frac{1}{1.93\sqrt{2\pi}} \int_{0}^{x} e^{-\frac{1}{2}\left(\frac{X - 6.4}{1.93}\right)^2} dX\ . \qquad (17)$$

The resulting cumulative distribution for the March probabilistic manpower requirement is presented in Figure 19.

The reader is again cautioned that the normal distribution should not be assumed for the probabilistic manpower requirement unless there are a large number of outstanding proposals. As with Model II, if conditions of the manpower planning problem do not support the assumption of normality, the

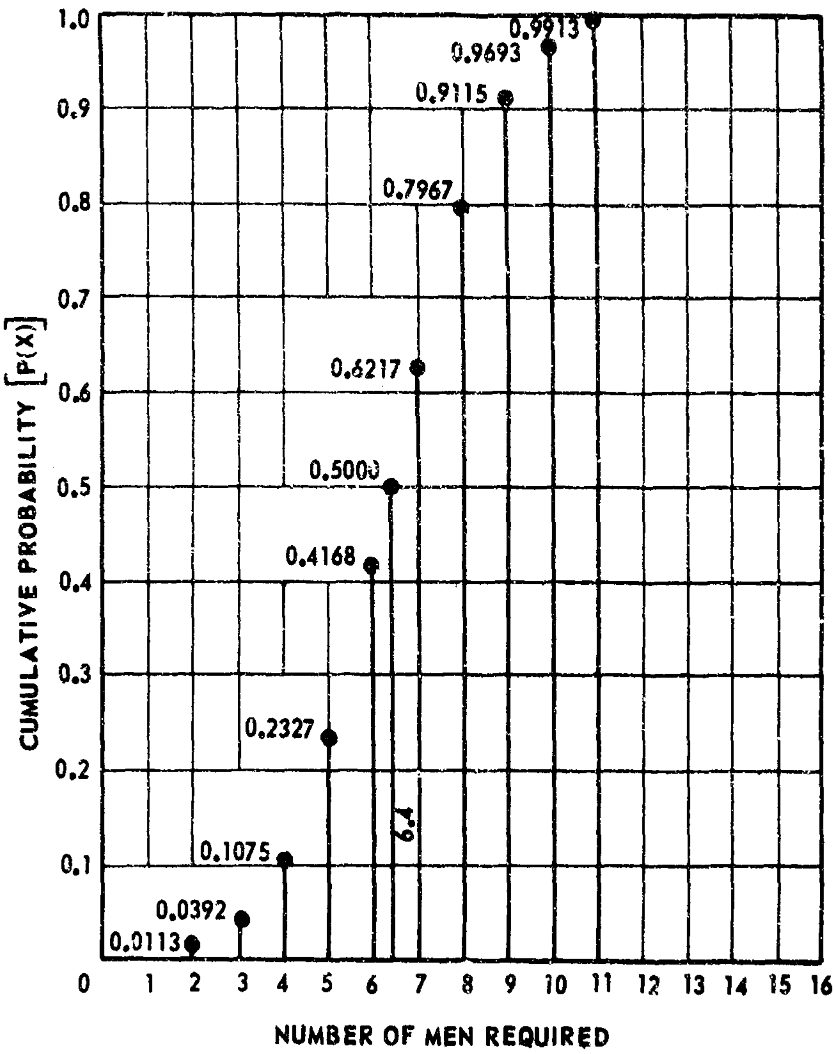

Figure 19. Cumulative Probabilistic Manpower Requirement

planning problem must be solved by a complete enumeration technique.

To develop a complete enumeration technique for Model III, reference is made to the enumeration process developed for Model II. It was shown that for one outstanding proposal there were two probable outcomes (Figure 20).

For proposal I, there is a p_1 probability that the contract will be received and that ${}_jM_1(t)$ men of type j will be required during the period t. There is also a p_1^* probability that the contract will not be received and zero

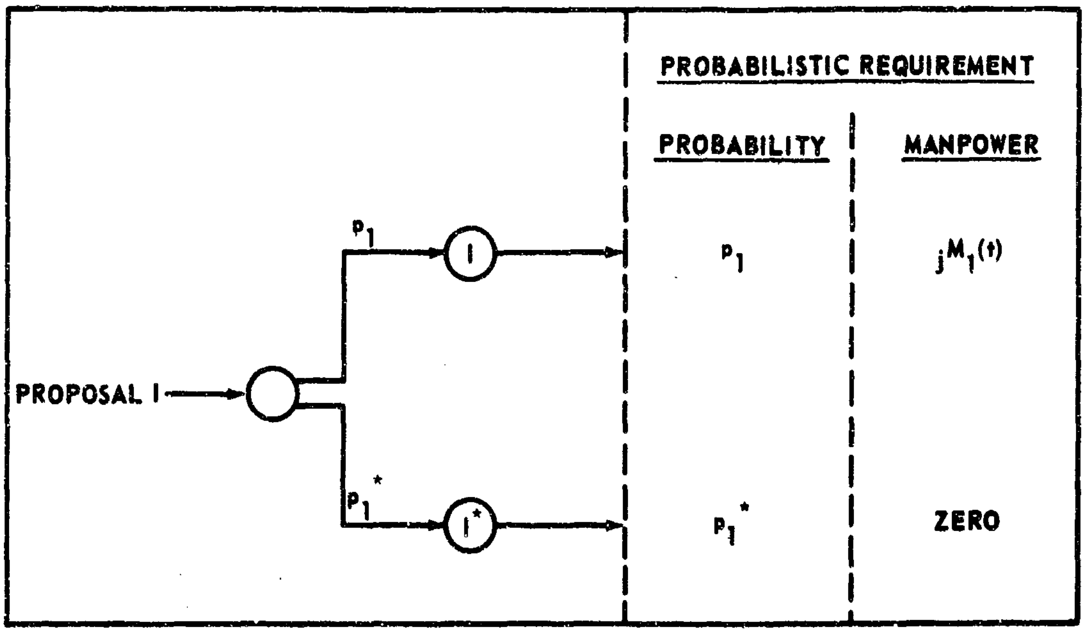

Figure 20. Enumeration Process for Model II: One Proposal

men will be required. For the case of two outstanding proposals, it may be shown for Model II that four outcomes are possible (Figure 21). The manpower requirements and the associated probability for each possible outcome are also presented in Figure 21. If the corporation has three proposals outstanding, it may be shown that eight outcomes are possible (Figure 22).

The technique developed for Model II may be expanded to include the probability of contract initiation date (Model III). For example, if one contract proposal is outstanding with a p_1 probability of capture and a $p_{t/1}$ conditional probability associated with the v_1 probable starting dates, it can be shown that $v_1 + 1$ outcomes are possible (Figure 23). Figure 23 represents the complete enumeration process for one outstanding proposal, where

t is the time period for which a probabilistic manpower requirement is to be determined.

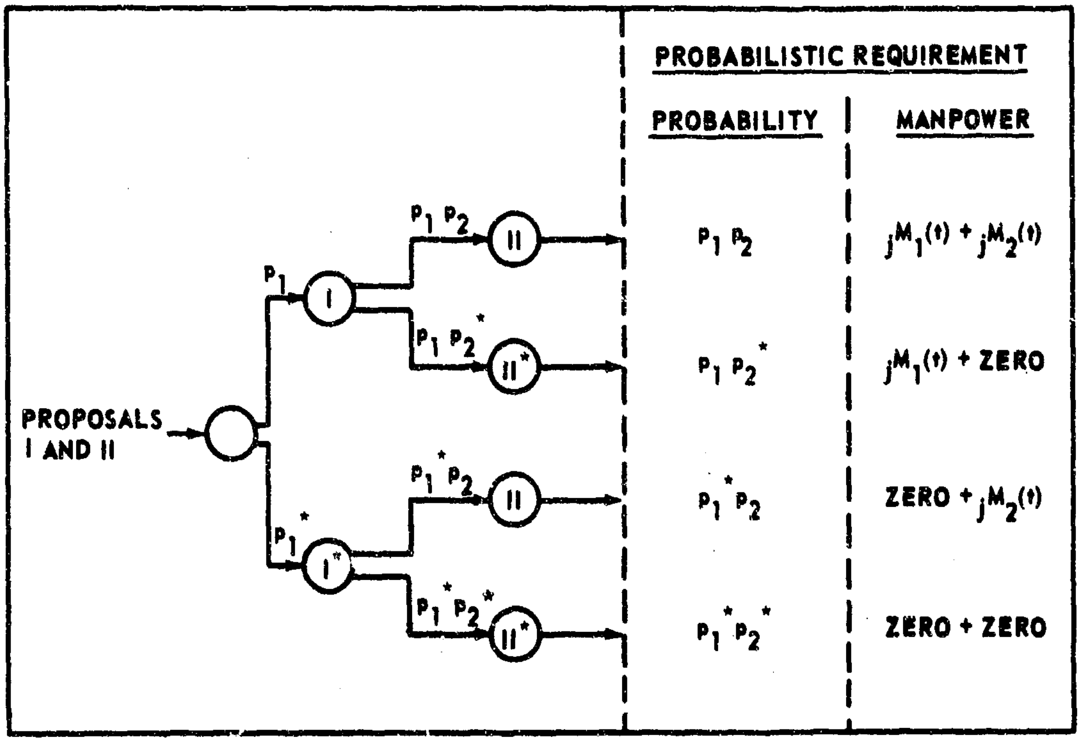

Figure 21. Enumeration Process for Model II: Two Proposals

p_1 is the probability of capture for project I.

p_1^* is the probability that project I will not be received ($p_1^* = 1 - p_1$).

t_1 is the first time period in which contract I may be initiated.

t_2 is the second time period in which contract I may be initiated.

t_{v_1} is the final time period in which contract I may be initiated.

v_1 is the total time period within which project I must be initiated if it is received.

$p_{t_1/1}$ is the probability of contract I being initiated in period t_1, given that contract I will be received.

$p_{t_2/1}$ is the probability that contract I will be initiated in period t_2, given that contract I will be received.

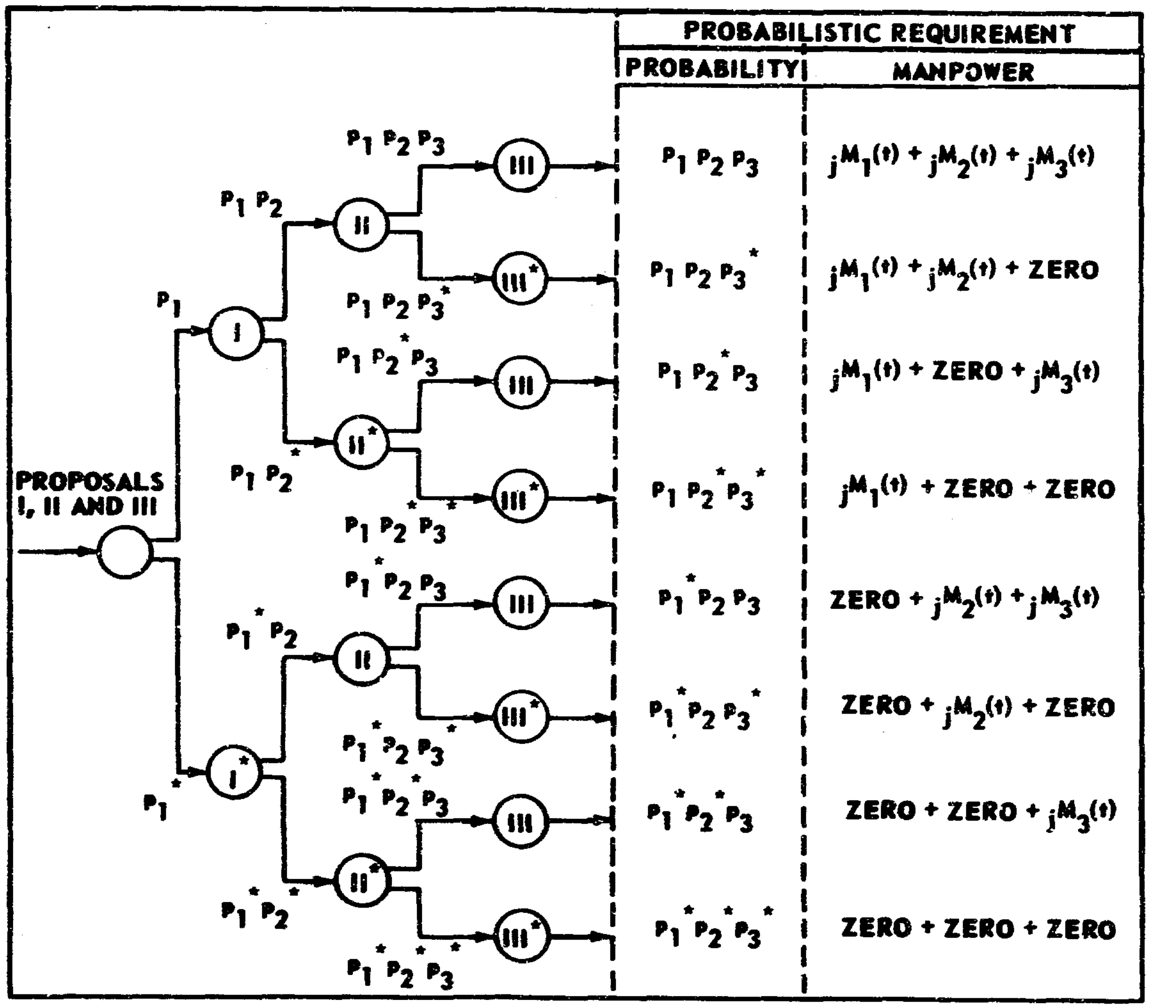

Figure 22. Enumeration Process for Model II: Three Proposals

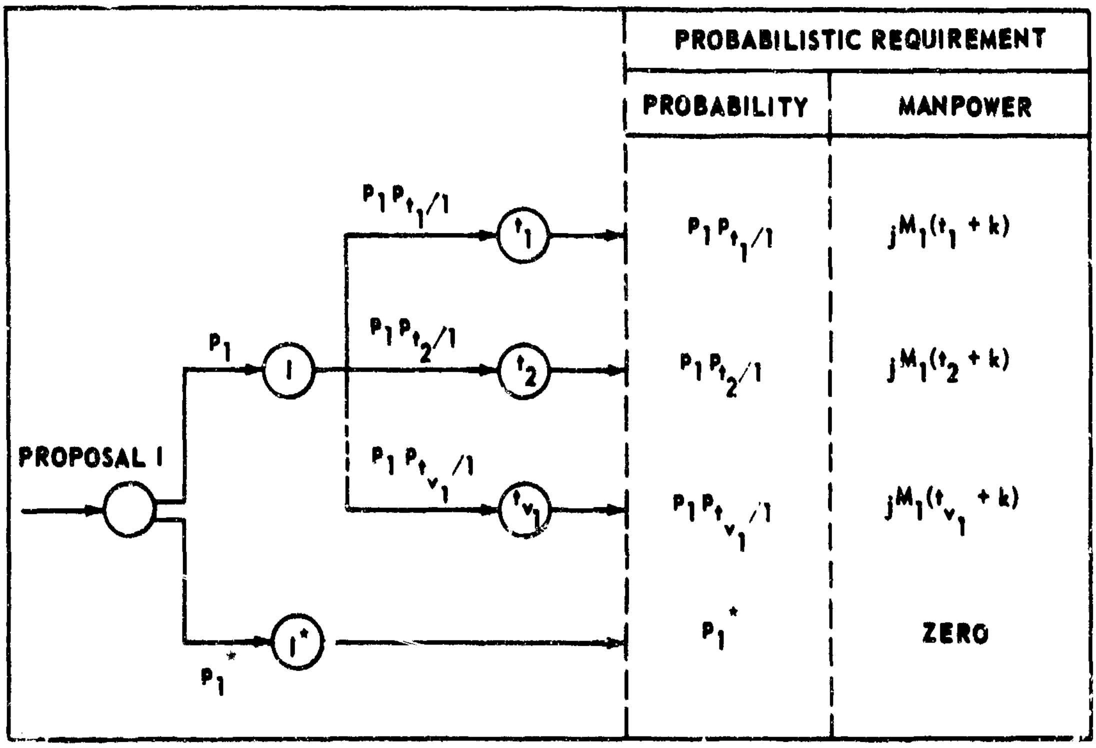

Figure 23. Enumeration Process for Model III: One Proposal

$p_{t_{v_1}/1}$ is the probability that contract I will be initiated in period t_{v_1}, given that contract I will be received.

$$\sum_{v=1}^{v_1} p_{t_v} = 1.$$

k is a variable depending on the time period of interest (t) and the assumed initiation period for the project.

$_jM_1\ (t_1 + k)$ is the number of type j men required during period t when it is assumed that project i will be initiated during period t_1.

$_jM_1\ (t_2 + k)$ is the number of type j men required during period t when it is assumed that project I will be initiated during period t_2.

$_jM_1\left(t_{v_1} + k\right)$ is the number of type j men required during period t when

it is assumed that project I will be initiated during period t_{v_1}.

As an illustration, consider the previous problem where manpower requirements and probable contract initiation dates were presented in Figures 12 and 13 respectively. Then, by using the enumeration notation,

$t =$ April (assumed)

$p_1 = 1$ (given condition)

$t_1 =$ January

$t_2 = t_{v_1} =$ February

$v_1 = 2$

$p_{t_1/1} = 0.6$

$p_{t_2/1} = p_{t_{v_1}/1} = 0.4$

${}_jM_1\ (t_1 + k) = 10$ (from Figure 12 where $k = 3\text{-}4$)

${}_jM_1\ (t_2 + k) = {}_jM_2\left(t_{v_1} + k\right) = 8$ (from Figure 12 where $k = 2\text{-}3$).

Application of these data to the enumeration process presented in Figure 23 yields the April manpower requirements presented in Table 12. It should be noted that results obtained by the enumeration technique are identical to those obtained by the previous method and presented in Table 11.

Table 12. Probabilistic Manpower Requirement for Project I in April

Probability		Cumulative	Manpower Requirement		Expected Requirement
Calculated					
$p_1 p_{t_1/1}$	0.6	0.6	${}_jM_1\ (t_1 + k)$	10	6.0
$p_1 p_{t_2/1}$	0.4	1.0	${}_jM_1\ (t_2 + k)$	8	3.2
p_1^*	0	1.0	zero	0	0.0
Total expected manpower requirement					9.2

For the case of two outstanding contract proposals, the complete enumeration technique yields $v_1 + v_2 + v_1 v_2 + 1$ possible outcomes as indicated in Figure 24. Notation for Figure 24 is identical to that for Figure 23 with the following additions:

p_2 is the probability that contract II will be received.

p_2^* is the probability that contract II will not be received.

$p_{t_1/2}$ is the probability that contract II will be initiated in its respective t_1 time period.

$p_{t_2/2}$ is the probability that contract II will be initiated in its respective t_2 time period.

$p_{t_{v_2}/2}$ is the probability that contract II will be initiated in its respective t_{v_2} time period.

$$\sum_{v=1}^{v_2} p_{t_v} = 1 \ .$$

${}_jM_2\ (t_1 + k)$ is the number of type j men required during period t when it is assumed that contract II will be initiated during its respective t_1 period.

${}_jM_2\ (t_2 + k)$ is the number of type j men required during period t when it is assumed that contract II wil' be initiated during its respective t_2 period.

${}_jM_2\ (t_2 + k)$ is the number of type j men required during period t when it is assumed that contract II will be initiated during its respective t_2 period.

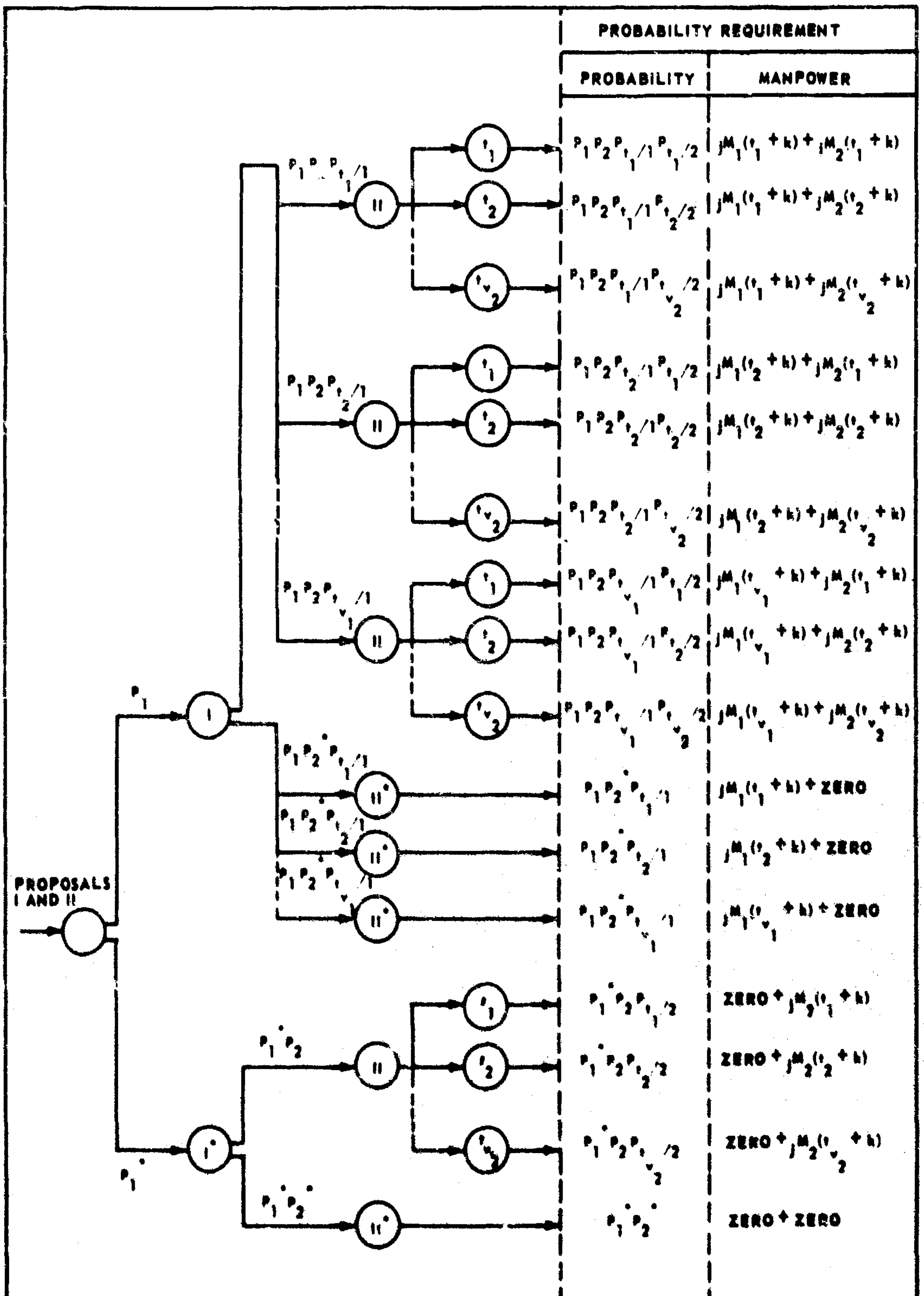

Figure 24. Enumeration Process for Model III: Two Proposals

${}_jM_2\,(t_{v_2} + k)$ is the number of type j men required during period t when it is assumed that contract II will be initiated during its respective t_{v_2} time period.

v_2 is the total time periods within which contract II must be initiated if it is received.

As the number of outstanding proposals increases, manpower estimation by complete enumeration becomes quite burdensome without the use of electronic computers. It can be shown that as the number of contracts increases from one to two, the enumerated outcomes increase from $v_1 + 1$ to $v_1 + v_2 + v_1 v_2 + 1$. If three contracts are considered, the number of enumeration outcomes increases to $v_1 + v_2 + v_3 + v_1 v_3 + v_2 v_3 + v_1 v_2 v_3 + 1$ (Figure 25). If a large number of proposals must be considered, and the assumption of normal distribution does not seem appropriate, a planning organization can usually determine the expected manpower requirements and the cumulative probability curves within acceptable tolerances by enumerating only a selected number of outcomes as suggested for Model II.

Fortunately, the complete enumeration process developed for Model III is a general solution to the problem of estimating future manpower requirements. Problems which inherently satisfy the requirements of Model I and Model II may be solved with the Model III enumeration technique by proper selection of the input subjective and conditional probabilities. The basic technique may also be expanded to include additional probabilistic variables. For example, it was previously indicated that the probability of meeting certain contractual milestones may be estimated by the utilization of Gantt and PERT

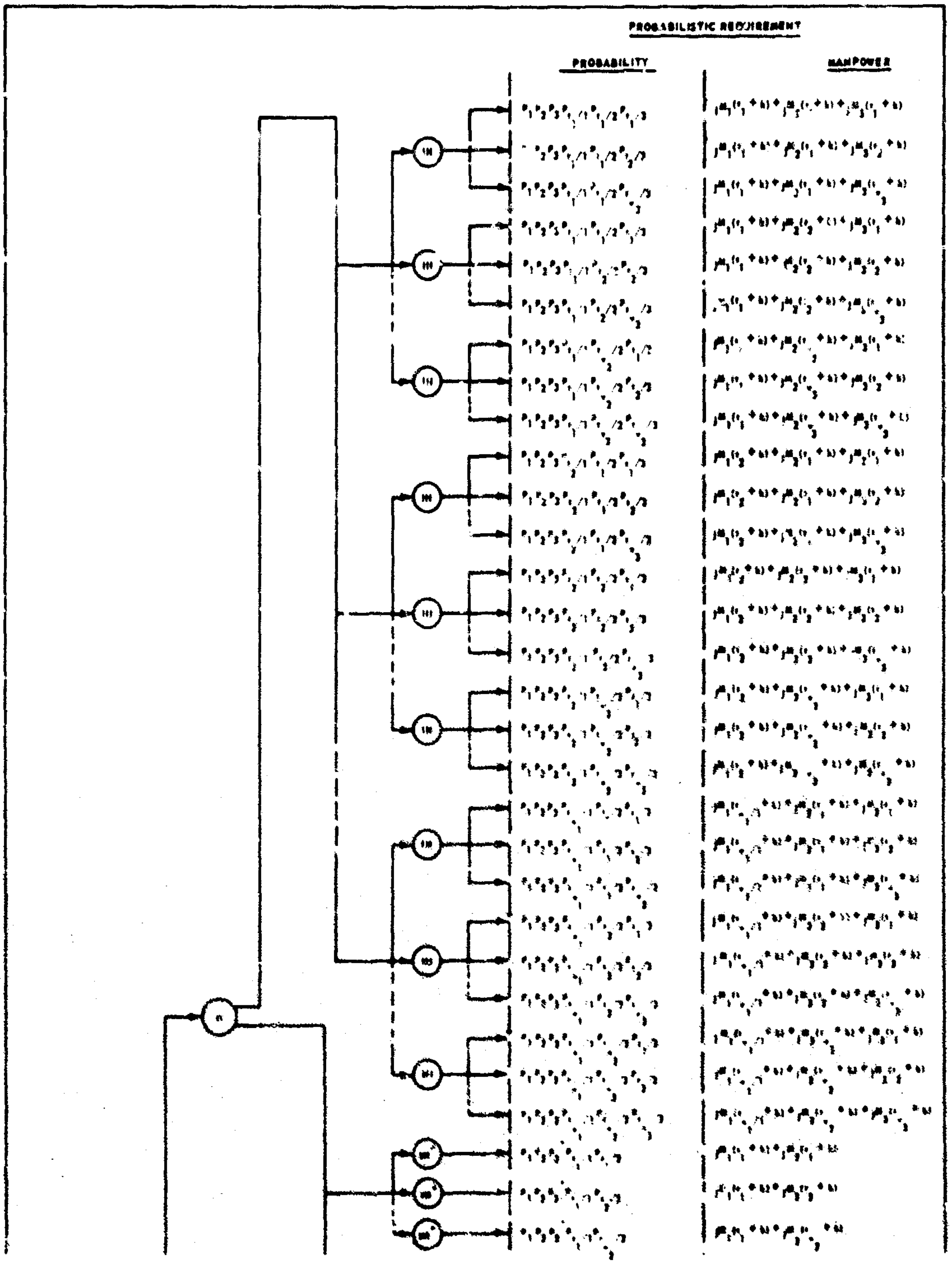

Figure 25. Enumeration Process for Model III: Three Proposals

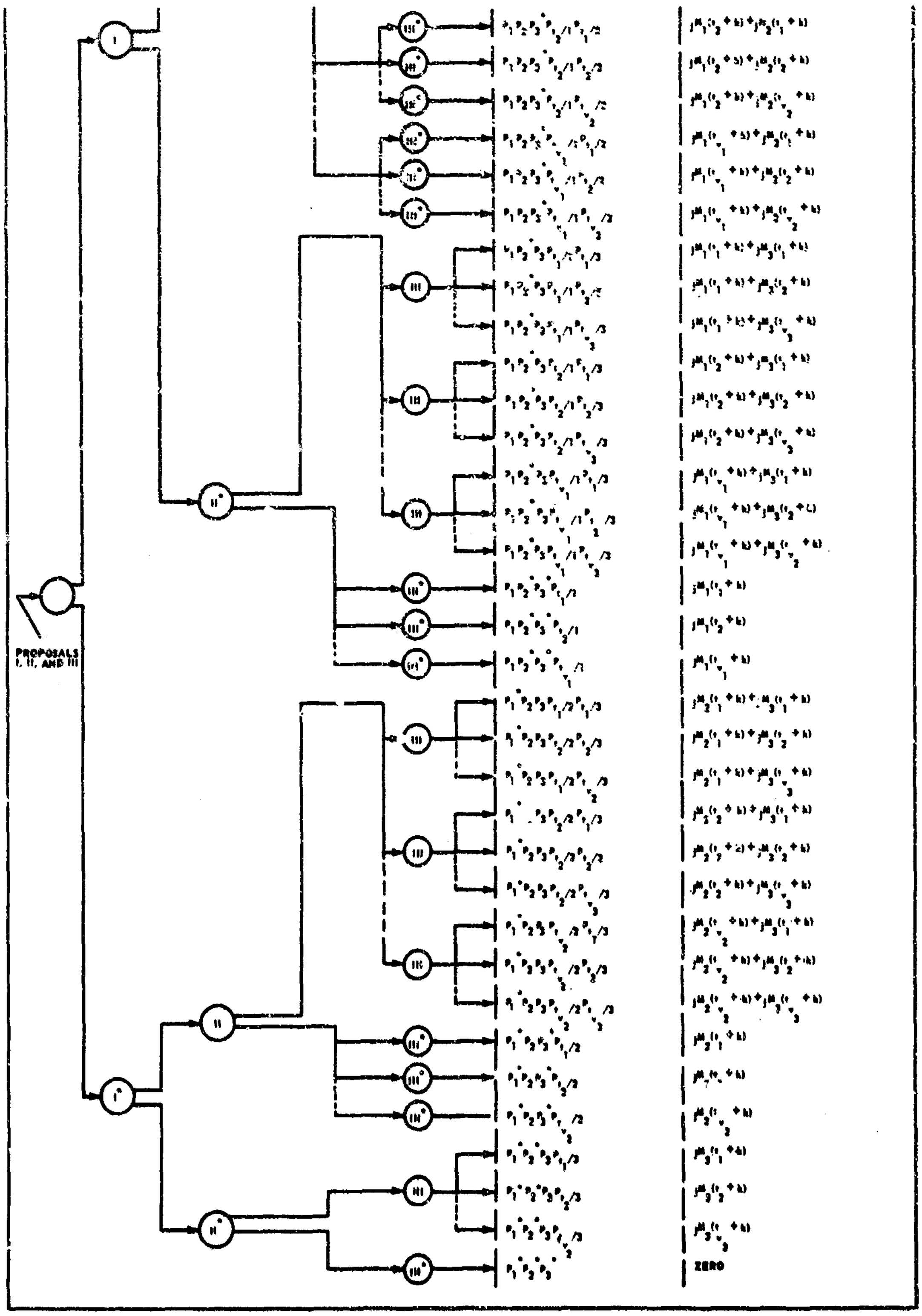

Figure 25. Enumeration Process for Model III: Three Proposals (Concluded)

scheduling techniques; therefore, Model III could be expanded to include the probabilistic program schedule.

In effort to clarify any question the reader may have concerning the mechanics of estimating future manpower requirements by complete enumeration, solution of a numerical problem with two outstanding proposals seems appropriate.

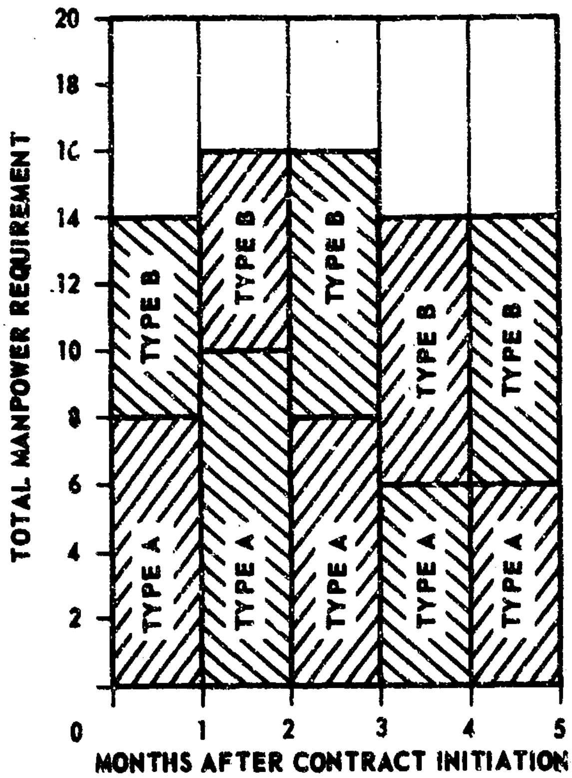

Figure 26. Time Adjusted Manpower Array for Contract I

Given two outstanding contract proposals with durations of five months and six months as shown in Figures 26 and 27, with probabilistic contract initiation dates as shown in Figures 28 and 29, and with capture probabilities of $p_1 = (0.8)$ and $p_2 = (0.7)$, what is the probabilistic manpower requirement? To solve this problem the probabilistic manpower requirement data presented in Figure 24 are utilized. The problem may be set up as shown in Tables 13 and 14 and solved as indicated in the Appendix. The resulting expected manpower requirement for these two outstanding proposals is presented in Figures 30 and 31. If management chooses to use some criteria other than expected values (80 percent confidence level for example), they may

utilize the actual distributions of the probabilistic manpower requirements as presented in the Appendix.

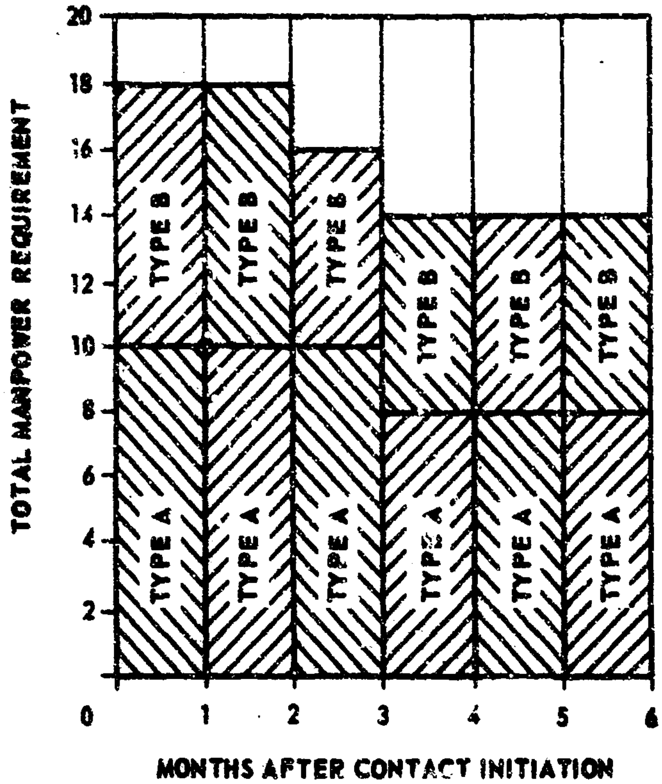

Figure 27. Time Adjusted Manpower Array for Contract II

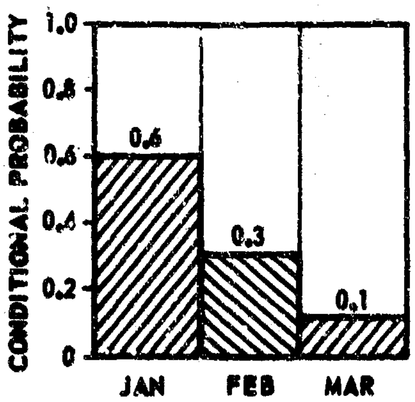

Figure 28. Probable Initiation Dates for Contract I

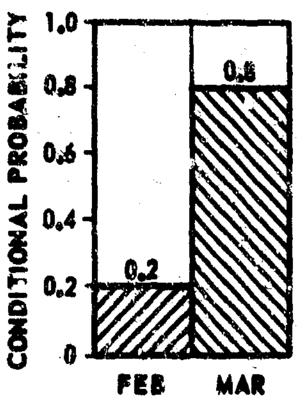

Figure 29. Probable Initiation Dates for Contract II

Table 13. Manpower Type A Probabilistic Requirements: Example Problem

Outcome Number	Probability of Outcome	General Equation	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
1	$p_1 p_2 p_{t_1/1} p_{t_1/2}$ = (0.8) (0.7) (0.6) (0.2) = 0.0672	${}_jM_1\ (t_1 + k) + {}_jM_2\ (t_1 + k)$	8 + 0 = 8	10 + 10 = 20	8 + 10 = 18	6 + 10 = 16	6 + 8 = 14	0 + 8 = 8	0 + 8 = 8	0 + 0 = 0	0 + 0 = 0
2	$p_1 p_2 p_{t_1/1} p_{t_2/2}$ = (0.8) (0.7) (0.6) (0.8) = 0.2688	${}_jM_1\ (t_1 + k) + {}_jM_2\ (t_2 + k)$	8 + 0 = 8	10 + 0 = 10	8 + 10 = 18	6 + 10 = 16	6 + 10 = 16	0 + 8 = 8	0 + 8 = 8	0 + 8 = 8	0 + 0 = 0
3	$p_1 p_2 p_{t_2/1} p_{t_1/2}$ = (0.8) (0.7) (0.3) (0.2) = 0.0336	${}_jM_1\ (t_2 + k) + {}_jM_2\ (t_1 + k)$	0 + 0 = 0	8 + 10 = 18	10 + 10 = 20	8 + 10 = 18	6 + 8 = 14	6 + 8 = 14	0 + 8 = 8	0 + 0 = 0	0 + 0 = 0
4	$p_1 p_2 p_{t_2/1} p_{t_2/2}$ = (0.8) (0.7) (0.3) (0.8) = 0.1344	${}_jM_1\ (t_2 + k) + {}_jM_2\ (t_2 + k)$	0 + 0 = 0	8 + 0 = 8	10 + 10 = 20	8 + 10 = 18	6 + 10 = 16	6 + 8 = 14	0 + 8 = 8	0 + 8 = 8	0 + 0 = 0
5	$p_1 p_2 p_{t_3/1} p_{t_1/2}$ = (0.8) (0.7) (0.1) (0.2) = 0.0112	${}_jM_1\ (t_3 + k) + {}_jM_2\ (t_1 + k)$	0 + 0 = 0	0 + 10 = 10	8 + 10 = 18	10 + 10 = 20	8 + 8 = 16	6 + 8 = 14	6 + 8 = 14	0 + 0 = 0	0 + 0 = 0
6	$p_1 p_2 p_{t_3/1} p_{t_2/2}$ = (0.8) (0.7) (0.1) (0.8) = 0.0448	${}_jM_1\ (t_3 + k) + {}_jM_2\ (t_2 + k)$	0 + 0 = 0	0 + 0 = 0	8 + 10 = 18	10 + 10 = 20	8 + 10 = 18	6 + 8 = 14	6 + 8 = 14	0 + 8 = 8	0 + 0 = 0
7	$p_1 p_2^* p_{t_1/1}$ = (0.8) (0.3) (0.6) = 0.1440	${}_jM_1\ (t_1 + k)$	8 = 8	10 = 10	8 = 8	6 = 6	6 = 6	0 = 0	0 = 0	0 = 0	0 = 0
8	$p_1 p_2^* p_{t_2/1}$ = (0.8) (0.3) (0.3) = 0.0720	${}_jM_1\ (t_2 + k)$	0 = 0	8 = 8	10 = 10	8 = 8	6 = 6	6 = 6	0 = 0	0 = 0	0 = 0
9	$p_1 p_2^* p_{t_3/1}$ = (0.8) (0.3) (0.1) = 0.0240	${}_jM_1\ (t_3 + k)$	0 = 0	0 = 0	8 = 8	10 = 10	8 = 8	6 = 6	6 = 6	0 = 0	0 = 0
10	$p_1^* p_2 p_{t_1/2}$ = (0.2) (0.7) (0.2) = 0.0280	${}_jM_2\ (t_1 + k)$	0 = 0	10 = 10	10 = 10	10 = 10	8 = 8	8 = 8	8 = 8	0 = 0	0 = 0
11	$p_1^* p_2 p_{t_2/2}$ = (0.2) (0.7) (0.8) = 0.1120	${}_jM_2\ (t_2 + k)$	0 = 0	0 = 0	10 = 10	10 = 10	10 = 10	8 = 8	8 = 8	8 = 8	0 = 0
12	$p_1^* p_2^*$ = (0.2) (0.3) = 0.0600	0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0

Table 14. Manpower Type B Probabilistic Requirements: Example Problem

Outcome Number	Probability of Outcome	General Equation	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
1	$p_1p_2P_{t_1/1}P_{t_1/2}$ = (0.8) (0.7) (0.6) (0.2) = 0.0672	$_jM_1\ (t_1+k) + {_jM_2}\ (t_1+k)$	6 + 0 = 6	6 + 8 = 14	8 + 8 = 16	8 + 6 = 14	8 + 6 = 14	0 + 6 = 6	0 + 6 = 6	0 + 0 = 0	0 + 0 = 0
2	$p_1p_2P_{t_1/1}P_{t_2/2}$ = (0.8) (0.7) (0.6) (0.8) = 0.2688	$_jM_1\ (t_1+k) + {_jM_2}\ (t_2+k)$	6 + 0 = 6	6 + 0 = 6	8 + 8 = 16	8 + 8 = 16	8 + 6 = 14	0 + 6 = 6	0 + 6 = 6	0 + 6 = 6	0 + 0 = 0
3	$p_1p_2P_{t_2/1}P_{t_1/2}$ = (0.8) (0.7) (0.3) (0.2) = 0.0336	$_jM_1\ (t_2+k) + {_jM_2}\ (t_1+k)$	0 + 0 = 0	6 + 8 = 14	6 + 8 = 14	8 + 6 = 14	8 + 6 = 14	8 + 6 = 14	0 + 6 = 6	0 + 0 = 0	0 + 0 = 0
4	$p_1p_2P_{t_2/1}P_{t_2/2}$ = (0.8) (0.7) (0.3) (0.8) = 0.1344	$_jM_1\ (t_2+k) + {_jM_2}\ (t_2+k)$	0 + 0 = 0	6 + 0 = 6	6 + 8 = 14	8 + 8 = 16	8 + 6 = 14	8 + 6 = 14	0 + 6 = 6	0 + 6 = 6	0 + 0 = 0
5	$p_1p_2P_{t_3/1}P_{t_1/2}$ = (0.8) (0.7) (0.1) (0.2) = 0.0112	$_jM_1\ (t_3+k) + {_jM_2}\ (t_1+k)$	0 + 0 = 0	0 + 8 = 8	6 + 8 = 14	6 + 6 = 12	8 + 6 = 14	8 + 6 = 14	8 + 6 = 14	0 + 0 = 0	0 + 0 = 0
6	$p_1p_2P_{t_3/1}P_{t_2/2}$ = (0.8) (0.7) (0.1) (0.8) = 0.0448	$_jM_1\ (t_3+k) + {_jM_2}\ (t_2+k)$	0 + 0 = 0	0 + 0 = 0	6 + 8 = 14	6 + 8 = 14	8 + 6 = 14	8 + 6 = 14	8 + 6 = 14	0 + 6 = 6	0 + 0 = 0
7	$p_1p_2^*P_{t_1/1}$ = (0.8) (0.3) (0.6) = 0.1440	$_jM_1\ (t_1+k)$	6 = 6	6 = 6	8 = 8	8 = 8	8 = 8	0 = 0	0 = 0	0 = 0	0 = 0
8	$p_1p_2^*P_{t_2/1}$ = (0.8) (0.3) (0.3) = 0.0720	$_jM_1\ (t_2+k)$	0 = 0	6 = 6	6 = 6	8 = 8	8 = 8	8 = 8	0 = 0	0 = 0	0 = 0
9	$p_1p_2^*P_{t_3/1}$ = (0.8) (0.3) (0.1) = 0.0240	$_jM_1\ (t_3+k)$	0 = 0	0 = 0	6 = 6	6 = 6	8 = 8	8 = 8	8 = 8	0 = 0	0 = 0
10	$p_1^*p_2P_{t_1/2}$ = (0.2) (0.7) (0.2) = 0.0280	$_jM_2\ (t_1+k)$	0 = 0	8 = 8	8 = 8	6 = 6	6 = 6	6 = 6	6 = 6	0 = 0	0 = 0
11	$p_1^*p_2P_{t_2/2}$ = (0.2) (0.7) (0.8) = 0.1120	$_jM_2\ (t_2+k)$	0 = 0	0 = 0	8 = 8	8 = 8	6 = 6	6 = 6	6 = 6	6 = 6	0 = 0
12	$p_1^*p_2^*$ = (0.2) (0.3) = 0.0600	0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0

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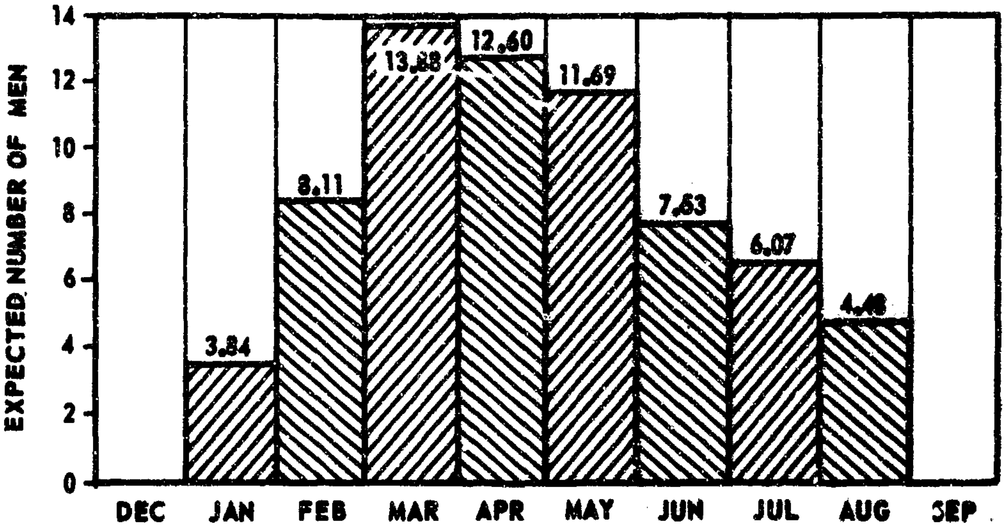

Figure 30. Corporation Expected Manpower Requirement: Type A

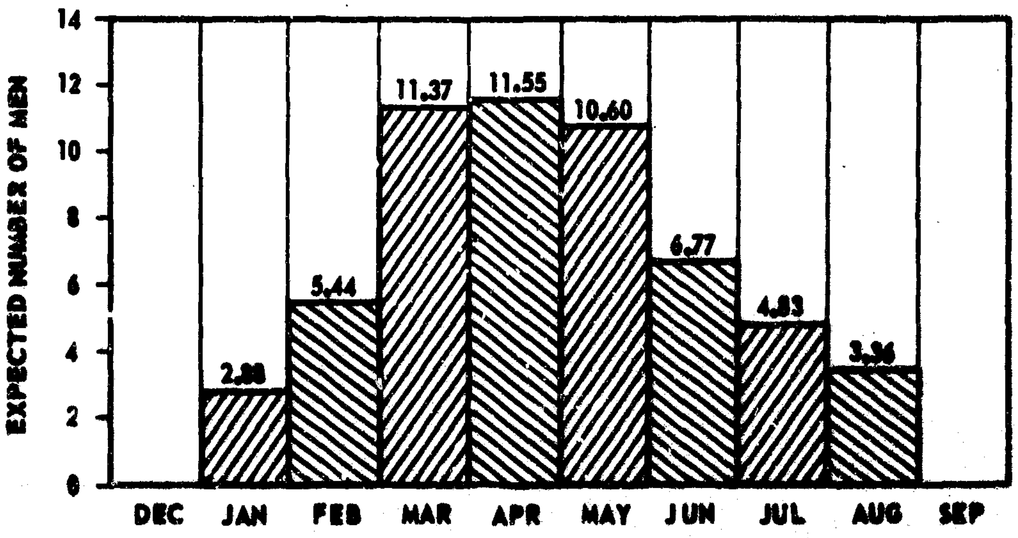

Figure 31. Corporation Expected Manpower Requirement: Type B

CHAPTER III

MINIMUM COST METHOD

A. Statement of the Problem

Given a finite work schedule, what are the optimum levels of full time employment for the corporation during the subject scheduling period? To evaluate this problem, it is necessary to recognize that hiring and training of employees prior to contract award can, in most instances, result in substantial reductions in costs by minimizing premium cost overtime. Also, reductions in the work force have tangible and intangible costs which the corporation is usually reluctant to incur. The problem then is to develop a strategy which optimizes full time employment and minimizes labor costs during the contract period consistent with company labor policies, union agreements, etc.

For application of the mathematical expressions developed in this chapter, the assumptions outlined in Chapter II, B and the procedures for developing contract schedules and associated manpower requirements outlined in Chapter II, C, 1 are adopted. It is also assumed in this thesis that the contractual workload can be processed by three types of manpower: full time company employees, overtime, and subcontracting. Full time employees are considered as either experienced (incumbent) or new employees.

There are many factors which influence the full time work force, such as voluntary terminations, and are not considered in this presentation. However, the models developed herein are readily adaptable to those situations peculiar to any given corporation.

B. Data Inputs

1. Finite Time — Adjusted Workload. For any given contract, management must be able to identify the numbers and types of personnel required for the duration of the contract. For example, contract I may require type A employees as indicated in Table 15. The time-adjusted manpower requirements reflect the number of experienced full time employees needed to perform the workload associated with the contract (or contracts).

Table 15. Time-Adjusted Type A Manpower Requirements for Contract I

Time Period	January	February	March	April
Workload (men)	10	10	20	30

2. Efficiency Factors. The different types of manpower may vary in net output per man; therefore, efficiency factors must be based on previous contract experience where management acquired evidence that some types of manpower are more (or less) productive than others.

Total capacity for the corporation during any period of time is therefore given by

$$Z_t = \lambda_1 X_{1,t} + \lambda_2 X_{2,t} + \lambda_4 X_{4,t} + \lambda_5 X_{5,t} + \lambda_6 X_{6,t} , \qquad (18)$$

where

Z_t is the effective capacity of manpower available during period t.

$X_{1,t}$ is the number of experienced employees available during period t.

$X_{2,t}$ is the number of new employees available during period t.

$X_{4,t}$ is the number of overtime units which can be worked by experienced employees during period t.

$X_{5,t}$ is the number of overtime units which new employees can work during period t.

$X_{6,t}$ is the number of subcontract personnel available during period t.

λ_1 is the efficiency factor for experienced employees and is normally 1.00 since the time-adjusted manpower requirement is based on experienced employees.

λ_2 is the efficiency factor for new employees and is usually less than λ_1.

λ_4 is the efficiency factor for experienced employee overtime and is usually less than λ_1.

λ_5 is the efficiency factor for new employee overtime and is usually less than λ_1 and also λ_4.

λ_6 is the efficiency factor for subcontract personnel and is usually less than λ_1.

Note that $X_{3,t}$, which is the number of full time employees to be terminated during period t, is not included in equation (18).

3. Cost Factors. Operating cost for a corporation during any period of time is not only a function of the manpower plan but also of the unit cost for

each type of manpower utilized. Management must again rely on previous experience for estimating the unit costs associated with advertising for and hiring new employees and estimating the tangible and intangible costs associated with mandatory termination of full time employees. Costs associated with overtime and subcontracts are usually pre-determined by labor agreements and contract negotiations. For the purpose of this analysis, unit costs will be identified as follows:

C_1 is the cost per time period for experienced personnel.

C_2 is the cost per time period associated with new personnel.

C_3 is the cost associated with termination of full time employees.

C_4 is the cost per time period for experienced employee overtime.

C_5 is the cost per time period for new employee overtime.

C_6 is the cost per time period for subcontract personnel.

4. Constraints. Due to management policies, labor agreements, etc., constraints usually exist which limit the range of values for X_i, where $i = 1, 2 \ldots 6$. It is also necessary that the effective capacity of manpower $\left(Z_t\right)$ satisfy the estimated workload $\left(W_t\right)$. In some instances it may be more economical to maintain excess capacity; therefore,

$$Z_t \geq W_t . \tag{19}$$

C. Development of Solution

As previously noted, the problem is one of determining the optimum manpower schedule for a given workload when the manpower schedule is subject

to constraints imposed by management policies and labor agreements. The optimum manpower schedule is defined as that schedule which permits the corporation to operate for the duration of the contract (or contracts) with minimum labor costs.

Labor costs for the manpower schedule are given by

$$C_T = \sum_t \left(C_1X_{1,t} + C_2X_{2,t} + C_3X_{3,t} + C_4X_{4,t} + C_5X_{5,t} + C_6X_{6,t}\right) \quad , \tag{20}$$

where the allowable range of values for $X_{1,t}$, $X_{2,t} \ldots X_{6,t}$ are

$0 \le X_{1,t} \le$ the number of experienced employees available at the beginning of period t (not including $X_{3,t}$).

$0 \le X_{2,t} \le$ maximum allowable number of new employees.

$0 \le X_{3,t} \le$ maximum allowable number of employee terminations.

$0 \le X_{4,t} \le$ maximum allowable units of experienced employee overtime.

$0 \le X_{5,t} \le$ maximum allowable units of new employee overtime.

$0 \le X_{6,t} \le$ maximum allowable subcontract units.

The minimum values for X_i are established by equation (19) or

$$W_t \le \lambda_1X_{1,t} + \lambda_2X_{2,t} + \lambda_4X_{4,t} + \lambda_5X_{5,t} + \lambda_6X_{6,t} \; . \tag{21}$$

Assuming that new employees may be adequately trained in one time period, then

$$X_{1,t+1} = X_{1,t} + X_{2,t} - X_{3,t+1} \; . \tag{22}$$

To derive the minimum cost schedule, a linear programming approach will be used. Requirements and data inputs for the problem are receptive to

the Simplex Technique[1] for cost minimization. An illustrative problem will be solved to demonstrate the cost minimization method.

Suppose the scheduling period consists of four months and the initial number of experienced employees is 60. The expected workload is shown in Table 16, and personnel constraints exist such that

$$0 \le X_{1,t} \le 100$$
$$0 \le X_{2,t} \le 20$$
$$\le X_{3,t} \le 20$$
$$0 \le X_{4,t} \le 30$$
$$0 \le X_{5,t} \le 30$$
$$0 \le X_{6,t} \le 20$$

Table 16. Projected Workload for Minimum Cost Example Problem

Time Period (t)	(1) January	(2) February	(3) March	(4) April
Workload (men)	50	70	90	60

Unit costs and efficiency factors for the various types of manpower are presented in Table 17. The problem is to determine the minimum cost plan for the given workload.

Referring to the Simplex Technique for solution, the objective function is

$$C_T = 100X_{1,1} + 130X_{2,1} + 50X_{3,1} + 150X_{4,1} + 150X_{5,1} + 170X_{6,1}$$
$$+ 100X_{1,2} + 130X_{2,2} + 50X_{3,2} + 150X_{4,2} + 150X_{5,2} + 170X_{6,2}$$

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[1]C. W. Churchman, R. L. Ackoff, E. L. Arnoff, Introduction to Operations Research. John Wiley & Sons, Inc., New York, New York, 1963, p. 304.

Table 17. Unit Costs and Efficiency Data for Minimum Cost Example Problem

Unit Cost ($)	Identification	Efficiency Factor
$C_1 = 100$	Experienced employees	$\lambda_1 = 1.00$
$C_2 = 130$	New employees	$\lambda_2 = 0.50$
$C_3 = 50$	Mandatory terminations	
$C_4 = 150$	Experienced employee overtime	$\lambda_4 = 0.70$
$C_5 = 150$	New employee overtime	$\lambda_5 = 0.35$
$C_6 = 170$	Subcontract	$\lambda_6 = 0.80$

$$+ 100X_{1,3} + 130X_{2,3} + 50X_{3,3} + 150X_{4,3} + 150X_{5,3} + 170X_{6,3}$$

$$+ 100X_{1,4} + 130X_{2,4} + 50X_{3,4} + 150X_{4,4} + 150X_{5,4} + 170X_{6,4},$$

and the constraints, assuming that a new employee is experienced after one month, are

(1) $X_{1,1} + 0.5X_{2,1} + 0.7X_{4,1} + 0.35X_{5,1} + 0.8X_{6,1} \geq 50$

(2) $X_{1,2} + 0.5X_{2,2} + 0.7X_{4,2} + 0.35X_{5,2} + 0.8X_{6,2} \geq 70$

(3) $X_{1,3} + 0.5X_{2,3} + 0.7X_{4,3} + 0.35X_{5,3} + 0.8X_{6,3} \geq 90$

(4) $X_{1,4} + 0.5X_{2,4} + 0.7X_{4,4} + 0.35X_{5,4} + 0.8X_{6,4} \geq 60$

(5) $X_{1,1} \leq 60$ (initial state)

$$\left.\begin{array}{l}
(6)\ X_{1,1} + X_{3,1} = 60 \\
(7)\ X_{1,2} + X_{3,2} - X_{1,1} - X_{2,1} = 0 \\
(8)\ X_{1,3} + X_{3,3} - X_{1,2} - X_{2,2} = 0 \\
(9)\ X_{1,4} + X_{3,4} - X_{1,3} - X_{2,3} = 0
\end{array}\right] \text{— Experienced employees during } t$$

(10) $X_{1,1} \geq 50$

(11) $X_{1,2} = 50$

(12) $X_{1,2} \leq 100$	Experienced employee constraints
(13) $X_{1,3} \geq 50$	
(14) $X_{1,3} \leq 100$	
(15) $X_{1,4} \geq 50$	
(16) $X_{1,4} \leq 100$	
(17) $X_{2,1} \leq 20$	New employee constraints
(18) $X_{2,2} \leq 20$	
(19) $X_{2,3} \leq 20$	
(20) $X_{2,4} \leq 20$	
(21) $X_{3,1} \leq 20$	Mandatory termination constraints
(22) $X_{3,2} \leq 20$	
(23) $X_{3,3} \leq 20$	
(24) $X_{3,4} \leq 20$	
(25) $X_{4,1} \leq 30$	Experienced employee overtime constraints
(26) $X_{4,2} \leq 30$	
(27) $X_{4,3} \leq 30$	
(28) $X_{4,4} \leq 30$	
(29) $X_{5,1} \leq 30$	New employee overtime constraints
(30) $X_{5,2} \leq 30$	
(31) $X_{5,3} \leq 30$	
(32) $X_{5,4} \leq 30$	
(33) $X_{6,1} \leq 20$	Subcontract constraints .
(34) $X_{6,2} \leq 20$	
(35) $X_{6,3} \leq 20$	

(36) $X_{6,4} \leq 20$

By using the Simplex Technique to minimize the objective function, the minimum cost solution is

January	February	March	April
$X_{1,1} = 60$	$X_{1,2} = 60$	$X_{1,3} = 30$	$X_{1,4} = 60$
	$X_{2,2} = 20$	$X_{6,3} = 12.5$	$X_{3,4} = 20$

The minimum cost plan is, therefore, to retain the original 60 employees during January and February, hire 20 additional employees prior to 1 February, subcontract 12.5 additional man months in March, and terminate 20 full time employees in April.

The reader should note that there are solutions which reduce the total cost for any given period (t); however, there are no other solutions which reduce the cost over the entire planning interval, January through April.

It should be evident that the Minimum Cost Method presented herein can readily be adapted to planning situations peculiar to any corporation at any given time. This may be accomplished by modifying the objective function and constraints to meet the needs of the planning problems at hand.

CHAPTER IV

MINIMUM RISK METHOD

A. Statement of the Problem

Given the probabilistic manpower requirements, management needs a decision-making tool which allows the corporation to plan for some level of full time employment and adjust with minimum consequence to the actual workload when it occurs.[1] As noted in Chapter II, the array of all possible workloads and their probabilities of occurrence can be calculated. In Chapter III, a minimum cost technique was developed to indicate the optimum manpower plan for performing each of the possible workloads. The Minimum Risk Method developed herein provides management with a plan which facilitates transition to the required manpower level, regardless of which possible workload occurs, and enables the corporation to perform its contract agreements with optimum overall labor costs.

To consider all of the possible workloads the notion of risk is introduced where risk for a given workload is defined as the cost of making a transition, in later intervals, from a specific manpower level to the appropriate Minimum

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[1]R. F. Jewett, "A Minimum Risk Manpower Scheduling Technique," Management Science, Vol. 13, No. 10, June, 1967, p. B-579.

Cost level. Since the actual workload can only be addressed in a probabilistic manner, risk is expressed as a function of workload and the corporate planned manpower level.

Transition must be accomplished in harmony with the Minimum Cost Method constraints noted in Chapter III, and consequently, transition may require several time intervals.

If the planned manpower level is above the Minimum Cost level for the workload which occurs, transition costs stem from excess capacity and possibly layoffs of personnel. If the planned manpower level is below the Minimum Cost level for the workload which occurs, transition costs stem from overtime and possibly excess hiring of personnel.

In summary, risk is the cost of adjusting to the workload that actually occurs. Once the actual workload becomes known, an adjustment or transition is made and the appropriate Minimum Cost schedule is followed. Therefore, the problem is to identify a manpower level for planning which minimizes total risk for the given range of workload levels.

B. Data Inputs

In effect, the Minimum Risk Method utilizes manpower planning techniques developed in Chapter II and Chapter III. The data inputs for Minimum Risk are (1) the enumerated array of all possible workloads and the probability of occurrence for each and (2) the Minimum Cost plan for performing each of the possible workloads.

C. Development of Solution

As previously noted, the problem is to identify a manpower planning level which minimizes total risk for the given range of workload levels.

For application of the mathematical expressions developed in the following paragraphs, all assumptions outlined in Chapter II, B and Chapter III, A are adopted.

The risk for each possible workload level is the transition cost for adjusting from the planned manpower level to the actual workload when it occurs. If C_k^* is the minimum cost for the k^{th} workload and C_k' is the cost of adjusting from the planned manpower level in the minimum number of time periods and completing the k^{th} workload, then the risk (R) for the k^{th} workload is given by

$$R_k = C_k' - C_k^* . \qquad (23)$$

The expected risk (R') for each of the k workloads, by equation (1), is therefore

$$R_k' = p_k R_k = p_k \left(C_k' - C_k^*\right), \qquad (24)$$

where p_k is the probability of occurrence for the k^{th} workload.

The total risk $[R(\cdot)]$ for any manpower plan is given by

$$R(\cdot) = \sum_k R_k' = \sum_k p_k \left(C_k' - C_k^*\right) \quad . \qquad (25)$$

The problem now becomes one of determining manpower planning levels $X_{1,t}'$ and $X_{2,t}'$ which minimize $R(\cdot)$ without violating the constraints noted for the Minimum Cost plan (Chapter III).

To solve the problem, an enumeration process is utilized whereby $R(\cdot)$ is calculated for all feasible values of $X'_{1,1}$ and $X'_{2,1}$. The enumeration which yields the minimum $R(\cdot)$ defines the Minimum Risk plan. This technique is demonstrated in the following illustrative problem.

Suppose the scheduling period is 4 months and the initial number of experienced employees is 60. The corporation has two outstanding proposals for R&D contracts with estimated capture probabilities of 0.6 and 0.3 respectively and it will not be known until 1 January if the contracts will be received. The type A manpower requirements for each contract are presented in Table 18. The projected manpower requirement without consideration of the two new contracts is also shown in Table 18.

Table 18. Type A Manpower Requirements: Minimum Risk Problem

Identification of Manpower Requirements	Probability	January	February	March	April
Workload without new contracts	1.0	50	70	90	60
Contract I	0.6	10	10	20	30
Contract II	0.3	30	20	20	10

Due to company policies and labor agreements, constraints exist such that

$$0 \leq X_{1,t} \leq 100$$

$$0 \leq X_{2,t} \leq 20$$

$$0 \leq X_{3,t} \leq 20$$

$$0 \leq X_{4,t} \leq 30$$

$$0 \leq X_{5,t} \leq 30$$

$$0 \leq X_{6,t} \leq 20$$

Unit costs and efficiency factors for the various types of manpower are presented in Table 19. (Note that this problem, without new contracts, was solved in Chapter III to illustrate the Minimum Cost Method.) It is also assumed that new employees can be trained within one time period.

Table 19. Unit Costs and Efficiency Data for Minimum Risk Problem

Unit Costs ($)	Identification	Efficiency Factors
$C_1 = 100$	Experienced employees	$\lambda_1 = 1.00$
$C_2 = 130$	New employees	$\lambda_2 = 0.50$
$C_3 = 50$	Mandatory terminations	
$C_4 = 150$	Experienced employee overtime	$\lambda_4 = 0.70$
$C_5 = 150$	New employee overtime	$\lambda_5 = 0.35$
$C_6 = 170$	Subcontracts	$\lambda_6 = 0.80$

The problem is to determine values for $X'_{1,1}$, $X'_{2,1}$, etc., which minimize the total risk for this planning situation.

Referring to the complete enumeration technique, Chapter II, there are four possible workloads with probabilities of occurrence as calculated and shown in Table 20. The Minimum Cost plan for each of the four possible workloads, determined by the Simplex Technique of Chapter III, is presented in Table 21.

Since the primary interest for planning is full time employees, and the

Table 20. Enumerated Workload Requirements for Minimum Risk Problem

Possible Workloads	Probability of Occurrence	Manpower Requirements			
		January	February	March	April
$W_1 = M_1^* M_2^*$	$p_1^* p_2^* = 0.28$	50	70	90	60
$W_2 = M_1 M_2^*$	$p_1 p_2^* = 0.42$	60	80	110	90
$W_3 = M_1^* M_2$	$p_1^* p_2 = 0.12$	80	90	110	70
$W_4 = M_1 M_2$	$p_1 p_2 = 0.18$	90	100	130	100

Table 21. Minimum Cost Manpower Plans for Minimum Risk Problem

Minimum Cost Plan for	Probability	January	February	March	April	Total Cost ($)
W_1	0.28	$X_{1,1} = 60$	$X_{1,2} = 60$ $X_{2,2} = 20$	$X_{1,3} = 80$ $X_{6,3} = 12.5$	$X_{1,4} = 60$ $X_{3,4} = 20$	31,725
W_2	0.42	$X_{1,1} = 60$ $X_{2,1} = 10$	$X_{1,2} = 70$ $X_{2,2} = 20$	$X_{1,3} = 90$ $X_{4,3} = 5.71$ $X_{6,3} = 20$	$X_{1,4} = 90$	39,157
W_3	0.12	$X_{1,1} = 60$ $X_{2,1} = 20$ $X_{6,1} = 12.5$	$X_{1,2} = 80$ $X_{2,2} = 10$ $X_{6,2} = 6.25$	$X_{1,3} = 90$ $X_{4,3} = 5.71$ $X_{6,3} = 20$	$X_{1,4} = 70$ $X_{3,4} = 20$	42,344
W_4	0.18	$X_{1,1} = 60$ $X_{2,1} = 20$ $X_{4,1} = 5.71$ $X_{6,1} = 20$	$X_{1,2} = 80$ $X_{2,2} = 20$ $X_{6,2} = 12.5$	$X_{1,3} = 100$ $X_{4,3} = 20$ $X_{6,3} = 20$	$X_{1,4} = 100$	51,982

Minimum Cost plans of Table 21 indicate that portions of the workload should be subcontracted, the workload and Minimum Cost arrays are modified to reflect only the in-house efforts. The in-house efforts are determined by subtracting the subcontracts from tables, and results of this operation are presented in Tables 22 and 23.

Table 22. In-House Workload Requirements for Minimum Risk Problem

Possible In-House Workloads	Probability of Occurrence	January	February	March	April
W'_1	0.28	50	70	80	60
W'_2	0.42	60	80	94	90
W'_3	0.12	70	85	94	70
W'_4	0.18	74	90	114	100

Table 23. Minimum Cost Plans for In-House Workloads

Minimum Cost Plan for	Probability	January	February	March	April	Total Cost ($)
W'_1	0.28	$X_{1,1} = 60$	$X_{1,2} = 60$ $X_{2,2} = 20$	$X_{1,3} = 80$	$X_{1,4} = 60$ $X_{3,4} = 20$	29,600
W'_2	0.42	$X_{1,1} = 60$ $X_{2,1} = 10$	$X_{1,2} = 70$ $X_{2,2} = 20$	$X_{1,3} = 90$ $X_{4,3} = 5.71$	$X_{1,4} = 90$	35,757
W'_3	0.12	$X_{1,1} = 60$ $X_{2,1} = 20$	$X_{1,2} = 80$ $X_{2,2} = 10$	$X_{1,3} = 90$ $X_{4,3} = 5.71$	$X_{1,4} = 70$ $X_{3,4} = 20$	35,757
W'_4	0.18	$X_{1,1} = 60$ $X_{2,1} = 20$ $X_{4,1} = 5.71$	$X_{1,2} = 80$ $X_{2,2} = 20$	$X_{1,3} = 100$ $X_{4,3} = 20$	$X_{1,4} = 100$	43,057

The next procedure is to enumerate costs and risks for all feasible values of $X'_{1,1}$ and $X'_{2,1}$ so that the minimum $R(\cdot)$ can be identified. It may be noted in Table 23 that $X_{1,1}$ is 60 for all four workloads; therefore, there is only one feasible solution for $X'_{1,1}$. However, $X_{2,1}$ varies from 0 to 20. The problem then is to determine a value for $X'_{2,1}$ which minimizes $R(\cdot)$ keeping in mind that the objective is to adjust to the Minimum Cost plan in the most expedient and economic manner consistent with the manpower constraints.

The results of the enumeration process for each of the four possible workloads is presented in Tables 24 through 27 where $X'_{2,1}$ was varied from 0 to 20 in increments of 5 units. Increments of five units each were arbitrarily selected for simplification of calculations in this illustrative problem. Note the heavy line in each of the tables. This line indicates when the level of full time employment reaches the Minimum Cost plan for that particular workload. Total cost for each of the trial solutions is also presented in Tables 24 through 27.

Summary of the expected risk $\left(R'_k\right)$ calculations by equation (24) with the cost data from Tables 24 through 27 is presented in Table 28. The term $R(\cdot)$ may then be calculated by equation (25) for each of the $X'_{2,1}$ solutions. The $R(\cdot)$ data are tabulated in Table 29 where it is shown that $R(\cdot)$ is minimum when $X'_{2,1}$ is 10 units.

Table 24. Enumerated Risk Plan for Workload Without New Contracts

	Period		January	February	March	April	Cost
	W'_i		50	70	80	60	C'_i
Solution	$X'_{1,1}$	$X'_{2,1}$					(\$)
$S_{1,1}$	60	0	$X_{1,1} = 60$	$X_{1,2} = 60$ $X_{2,1} = 20$	$X_{1,3} = 80$	$X_{1,4} = 60$ $X_{3,4} = 20$	29,600
$S_{1,2}$	60	5	$X_{1,1} = 60$ $X_{2,1} = 5$	$X_{1,2} = 65$ $X_{2,2} = 15$	$X_{1,3} = 80$	$X_{1,4} = 60$ $X_{3,4} = 20$	30,100
$S_{1,3}$	60	10	$X_{1,1} = 60$ $X_{2,1} = 10$	$X_{1,2} = 70$ $X_{2,2} = 10$	$X_{1,3} = 80$	$X_{1,4} = 60$ $X_{3,4} = 20$	30,600
$S_{1,4}$	60	15	$X_{1,1} = 60$ $X_{2,1} = 15$	$X_{1,2} = 75$ $X_{2,2} = 5$	$X_{1,3} = 80$	$X_{1,4} = 60$ $X_{3,4} = 20$	31,100
$S_{1,5}$	60	20	$X_{1,1} = 60$ $X_{2,1} = 20$	$X_{1,2} = 80$	$X_{1,3} = 80$	$X_{1,4} = 60$ $X_{3,4} = 20$	31,600

Table 25. Fnumerated Risk Plan if Contract I is Received

Solution	Period W'_2		January	February	March	April	Cost
			60	80	94	90	C'_2
	$X'_{1,1}$	$X'_{2,1}$					($)
$S_{2,1}$	60	0	$X_{1,1} = 60$	$X_{1,2} = 60$ $X_{2,2} = 20$ $X_{4,2} = 14.3$	$X_{1,3} = 80$ $X_{2,3} = 10$ $X_{4,3} = 12.9$	$X_{1,4} = 90$	36,980
$S_{2,2}$	60	5	$X_{1,1} = 60$ $X_{2,1} = 5$	$X_{1,2} = 65$ $X_{2,2} = 20$ $X_{4,2} = 7.1$	$X_{1,3} = 85$ $X_{2,3} = 5$ $X_{4,3} = 9.3$	$X_{1,4} = 90$	36,360
$S_{2,3}$	60	10	$X_{1,1} = 60$ $X_{2,1} = 10$	$X_{1,2} = 70$ $X_{2,2} = 20$	$X_{1,3} = 90$ $X_{4,3} = 5.7$	$X_{1,4} = 90$	35,757
$S_{2,4}$	60	15	$X_{1,1} = 60$ $X_{2,1} = 15$	$X_{1,2} = 75$ $X_{2,2} = 15$	$X_{1,3} = 90$ $X_{4,3} = 5.7$	$X_{1,4} = 90$	36,255
$S_{2,5}$	60	20	$X_{1,1} = 60$ $X_{2,1} = 20$	$X_{1,2} = 80$ $X_{2,2} = 10$	$X_{1,3} = 90$ $X_{4,3} = 5.7$	$X_{1,4} = 90$	36,755

Table 26. Enumerated Risk Plan if Contract II is Received

Solution	Period W'_3		January	February	March	April	Cost
			70	85	94	70	C'_3
	$X'_{1,1}$	$X'_{2,1}$					($)
$S_{3,1}$	60	0	$X_{1,1} = 60$ $X_{4,1} = 14.3$	$X_{1,2} = 60$ $X_{2,2} = 20$ $X_{4,2} = 21.4$	$X_{1,3} = 80$ $X_{4,3} = 20$	$X_{1,4} = 70$ $X_{3,4} = 10$	38,455
$S_{3,2}$	60	5	$X_{1,1} = 60$ $X_{2,1} = 5$ $X_{4,1} = 10.7$	$X_{2,1} = 65$ $X_{2,2} = 20$ $X_{4,2} = 14.3$	$X_{1,3} = 85$ $X_{4,3} = 12.9$	$X_{1,4} = 70$ $X_{3,4} = 15$	37,685
$S_{3,3}$	60	10	$X_{1,1} = 60$ $X_{2,1} = 10$ $X_{4,1} = 7.1$	$X_{1,2} = 70$ $X_{2,2} = 20$ $X_{4,2} = 7.1$	$X_{1,3} = 90$ $X_{4,3} = 5.7$	$X_{1,4} = 70$ $X_{3,4} = 20$	36,917
$S_{3,4}$	60	15	$X_{1,1} = 60$ $X_{2,1} = 15$ $X_{4,1} = 3.6$	$X_{1,2} = 75$ $X_{2,2} = 15$ $X_{4,2} = 3.6$	$X_{1,3} = 90$ $X_{4,3} = 5.7$	$X_{1,4} = 70$ $X_{3,4} = 20$	36,335

Table 26. Enumerated Risk Plan if Contract II is Received (Concluded)

Solution	Period W'_3 $X'_{1,1}$	$X'_{2,1}$	January 70	February 85	March 94	April 70	Cost C'_3 (\$)
$S_{3,5}$	60	20	$X_{1,1} = 60$ $X_{2,1} = 20$	$X_{1,2} = 80$ $X_{2,2} = 10$	$X_{1,3} = 90$ $X_{4,3} = 5.7$	$X_{1,4} = 70$ $X_{3,4} = 20$	35,757

Table 27. Enumerated Risk Plan if Contracts I and II are Received

Solution	Period W'_4 $X'_{1,1}$	$X'_{2,1}$	January 74	February 90	March 114	April 100	Cost C'_4 (\$)
$S_{4,1}$	60	0	$X_{1,1} = 60$ $X_{4,1} = 20$	$X_{1,2} = 60$ $X_{2,2} = 20$ $X_{4,2} = 28.6$	$X_{1,3} = 80$ $X_{2,3} = 20$ $X_{4,3} = 30$ $X_{5,3} = 8.6$	$X_{1,4} = 100$	48,280
$S_{4,2}$	60	5	$X_{1,1} = 60$ $X_{2,1} = 5$ $X_{4,1} = 16.4$	$X_{1,2} = 65$ $X_{2,2} = 20$ $X_{4,2} = 21.4$	$X_{1,3} = 85$ $X_{2,3} = 15$ $X_{4,3} = 30$ $X_{5,3} = 1.4$	$X_{1,4} = 100$	46,580
$S_{4,3}$	60	10	$X_{1,1} = 60$ $X_{2,1} = 10$ $X_{4,1} = 12.9$	$X_{1,2} = 70$ $X_{2,2} = 20$ $X_{4,2} = 14.3$	$X_{1,3} = 90$ $X_{2,3} = 10$ $X_{4,3} = 27.1$	$X_{1,4} = 100$	45,345
$S_{4,4}$	60	15	$X_{1,1} = 60$ $X_{2,1} = 15$ $X_{4,1} = 9.3$	$X_{1,2} = 75$ $X_{2,2} = 20$ $X_{4,2} = 7.1$	$X_{1,3} = 95$ $X_{2,3} = 5$ $X_{4,3} = 23.6$	$X_{1,4} = 100$	44,200
$S_{4,5}$	60	20	$X_{1,1} = 60$ $X_{2,1} = 20$ $X_{4,1} = 5.7$	$X_{1,2} = 80$ $X_{2,2} = 20$	$X_{1,3} = 100$ $X_{4,3} = 20$	$X_{1,4} = 100$	43,057

Table 28. Risk Summary for Example Problem

Solution	p_k	C'_k	C^*_k	R_k	R'_k
$S_{1,1}$	0.28	29,600	29,600	0	0
$S_{1,2}$	0.28	30,100	29,600	500	140
$S_{1,3}$	0.28	30,600	29,600	1000	280

Table 28. Risk Summary for Example Problem (Concluded)

Solution	p_k	C_k'	C_k^*	R_k	R_k
$S_{1,4}$	0.28	31,000	29,600	1500	420
$S_{1,5}$	0.28	31,600	29,600	2000	560
$S_{2,1}$	0.42	36,980	35.757	1223	514
$S_{2,2}$	0.42	36,360	35,757	603	253
$S_{2,3}$	0.42	35,757	35,757	0	0
$S_{2,4}$	0.42	36,255	35,757	498	209
$S_{2,5}$	0.42	36,755	35,757	998	419
$S_{3,1}$	0.12	38,455	35,757	2698	324
$S_{3,2}$	0.12	37,685	35,757	1928	231
$S_{3,3}$	0.12	36,917	35,757	1160	139
$S_{3,4}$	0.12	36,335	35,757	578	69
$S_{3,5}$	0.12	35,757	35,757	0	0
$S_{4,1}$	0.18	48,280	43,057	5223	940
$S_{4,2}$	0.18	46,580	43,057	3523	634
$S_{4,3}$	0.18	45,345	43,057	2288	412
$S_{4,4}$	0.18	44,200	43,057	1143	206
$S_{4,5}$	0.18	43,057	43,057	0	0

Table 29. Risk Analysis for Example Problem

$X'_{1,1}$	$X'_{2,1}$	ΣS	ΣR_k	$R(\cdot)$
60	0	$S_{1,1}+S_{2,1}+S_{3,1}+S_{4,1}$	0 + 514 + 324 + 940	1778
60	5	$S_{1,2}+S_{2,2}+S_{3,2}+S_{4,2}$	140 + 253 + 231 + 634	1258
60	10	$S_{1,3}+S_{2,3}+S_{3,3}+S_{4,3}$	280 + 0 + 139 + 412	831
60	15	$S_{1,4}+S_{2,4}+S_{3,4}+S_{4,4}$	420 + 209 + 69 + 206	904
60	20	$S_{1,5}+S_{2,5}+S_{3,5}+S_{4,5}$	560 + 419 + 0 + 0	979

What does this solution mean? The Minimum Risk plan for this illustrative problem is to retain the 60 experienced employees who will be available for January and, in addition, hire 10 new employees before January so that their services will be available during the first month. If workload W_1 occurs, the

corporate plan will be as shown in Table 30. If workloads W_2, W_3, or W_4 occur, the corporate plan will be as shown in Tables 31 through 33 respectively.

Table 30. Minimum Risk Plan Without New Contracts

$X'_{1,1}$	$X'_{2,1}$	January	February	March	April
60	10	$X_{1,1} = 60$	$X_{1,2} = 70$	$X_{1,3} = 80$	$X_{1,4} = 60$
		$X_{2,1} = 10$	$X_{2,2} = 10$	$X_{6,3} = 12.5$	$X_{3,4} = 20$

Table 31. Minimum Risk Plan if Contract I is Received

$X'_{1,1}$	$X'_{2,1}$	January	February	March	April
60	10	$X_{1,1} = 60$	$X_{1,2} = 70$	$X_{1,3} = 90$	$X_{1,4} = 90$
		$X_{2,1} = 10$	$X_{2,2} = 20$	$X_{4,3} = 5.7$	
				$X_{6,3} = 20$	

Table 32. Minimum Risk Plan if Contract II is Received

$X'_{1,1}$	$X'_{2,1}$	January	February	March	April
60	10	$X_{1,1} = 60$	$X_{1,2} = 70$	$X_{1,3} = 90$	$X_{1,4} = 70$
		$X_{2,1} = 10$	$X_{2,2} = 20$	$X_{4,3} = 5.7$	$X_{3,4} = 20$
		$X_{4,1} = 7.1$	$X_{4,2} = 7.1$	$X_{6,3} = 20$	
		$X_{6,1} = 12.5$	$X_{6,2} = 6.25$		

Table 33. Minimum Risk Plan if Contracts I and II are Received

$X'_{1,1}$	$X'_{2,1}$	January	February	March	April
60	10	$X_{1,1} = 60$	$X_{1,2} = 70$	$X_{1,3} = 90$	$X_{1,4} = 100$
		$X_{2,1} = 10$	$X_{2,2} = 20$	$X_{2,3} = 10$	
		$X_{4,1} = 12.9$	$X_{4,2} = 14.3$	$X_{4,3} = 27.1$	
		$X_{6,1} = 20$	$X_{6,2} = 12.5$	$X_{6,3} = 20$	

The corporation is hereby presented with a strategy for planning the future manpower requirements in the face of uncertainty. Note that no "pat" answer has been achieved; however, a mathematical simulation of the manpower planning system has been developed to assist management in understanding the status quo. Planning tools developed herein can be computerized and provide management with a rapid assessment of the situation at any given time. As new or more reliable data become available, the corporate plan can readily be modified to cope with the uncertain future.

As a point of interest and an illustration of the utility of the Minimum Risk Method, it is indicated in Figure 32 how $R(\cdot)$ varies with $X'_{2,1}$ for the previous example problem. It is evident that there is less risk for the corporation if planning error results in excess full time employees than if planning error results in too few employees. Information such as this may be very beneficial to the corporation management.

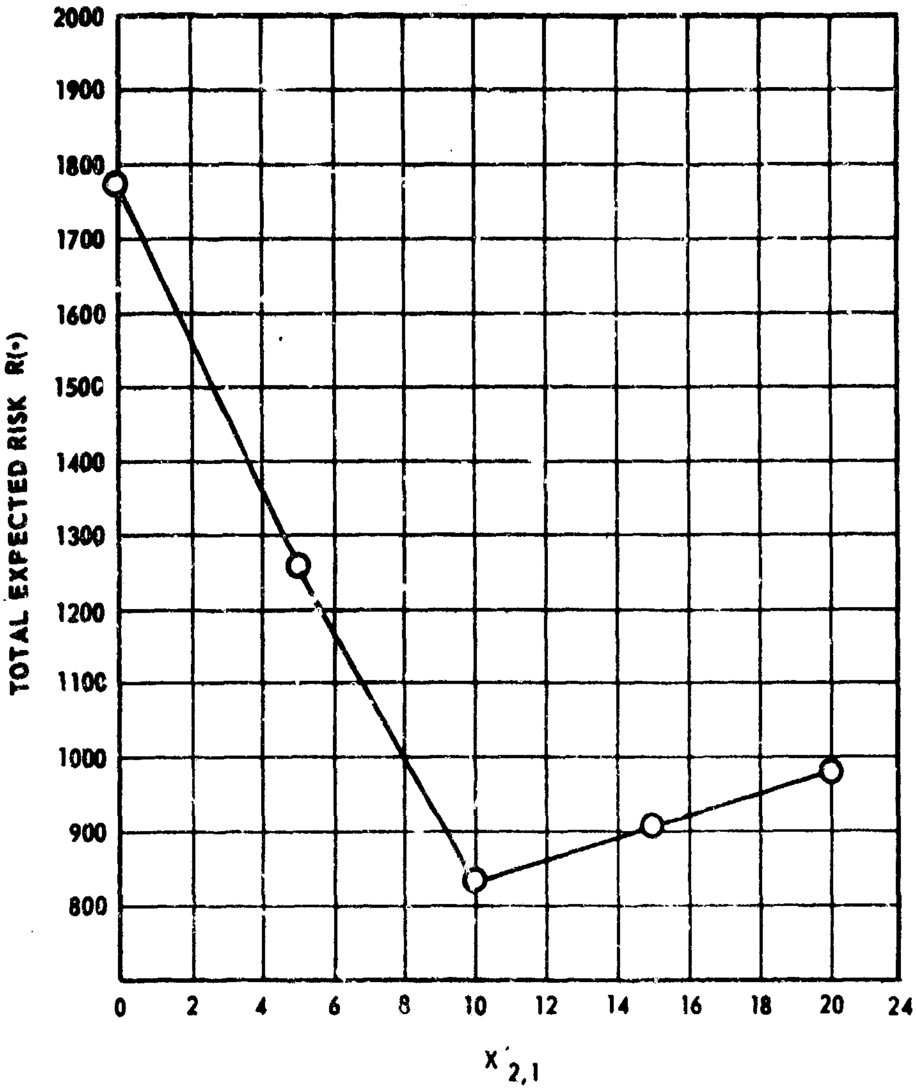

Figure 32. Total Risk Versus New Employees for Minimum Risk Example Problem

CHAPTER V

SUMMARY AND RECOMMENDATIONS

A. Summary

A corporation which bids competitively for government R&D contracts requires control systems and analytical planning tools that are somewhat unique because of the uncertainty associated with R&D tasks and the limited number of contracts open for proposals at any given time. Manpower requirements can vary considerably and planning techniques developed in the past for production type contracts, where the number of outstanding contract proposals may be large and the scopes of work are clearly defined, are totally inadequate.

For the purpose of this study, probabilistic manpower planning for R&D was visualized as a process for continually and systematically evaluating the manpower requirements and recognizing the risks being taken with the corporation's chosen manpower plan.

A technique was presented in Chapter II to provide management with a display of all possible workloads and an estimate of the probability of occurence for each. In Chapter III an optimization technique was developed to indicate the minimum cost manpower plan, consistent with company policies, labor agreements, etc., for each of the possible workloads. Finally, in Chapter IV, a technique was developed to provide management with a plan which allows the

corporate manpower level to be adjusted with minimum regret to the actual workload when it occurs; i.e., a manpower level is provided for planning such that the total expected risk associated with all of the possible workloads is minimized and there is a predetermined manpower plan for performing the actual   when it becomes known.

Mathematical models developed and presented are suitable for solution on electronic computers and, consequently, may provide management with a rapid evaluation of possible management decisions. Thus the corporation is provided with a mathematical simulation of the manpower planning system. The simulation can be used to readily evaluate effects of various inputs and promote better understanding of the problem at hand.

Use of these methods is limited only by the availability of accurate input data; therefore, for effective application, management must be prepared to establish and maintain a permanent organization for producing, accumulating, and processing the necessary data.

In short, this research has developed a powerful analytical manpower planning tool for management.

B. Recommendations for Future Research

In conducting this research the author became aware of several areas which degrade the effectiveness of probabilistic manpower planning through insufficient techniques for estimating and quantifying information. These areas are:

1. The estimation and quantification of subjective probabilities such as

probability for contract award and contract initiation date.

2. Identification of the types and numbers of manpower required to perform an R&D task in a given period of time.

3. Estimation of efficiency and cost factors for the various types of personnel.

Most of these problem areas have been researched as noted throughout this report, however, there remains a great need for improved techniques and all of these areas provide opportunity for future research.

It should be noted that the techniques developed consider only a fixed contract schedule. It is not uncommon for an R&D schedule to be extended because of technical difficulties encountered during the contract period. A profitable area for future research would be to modify the mathematical models of Chapter II to include this area of uncertainty and hopefully increase the versatility of the planning models. Both Gantt and PERT techniques allow planners to assess the probability of meeting given milestones on specific dates and should be of value in such a research program.

Because the probabilistic manpower requirements for different types of manpower must be calculated independently, it is possible that desired personnel ratios, such as engineers to technicians, will not be maintained for every contract and the corporation as a whole. If the efficiency factors and unit costs for the various types of manpower are proportional, the proper personnel ratios should be maintained. Otherwise, special techniques must be developed to assure that satisfactory personnel ratios are maintained. As a project for future research, it is recommended that the minimum cost and minimum risk

models of Chapters III and IV be amplified to include provisions maintaining acceptable personnel ratios in the linear programming operation.

APPENDIX

PROBABILISTIC MANPOWER EXAMPLE PROBLEM

Statement of the Problem

Given two outstanding contract proposals with durations of five months and six months respectively, with time adjusted manpower arrays as shown in Figures A-1 and A-2, with probabilistic contract initiation dates as shown in Figures A-3 and A-4, and capture probabilities of $p_1 = (0.8)$ and $p_2 = (0.7)$, what is the probabilistic manpower requirement versus time?

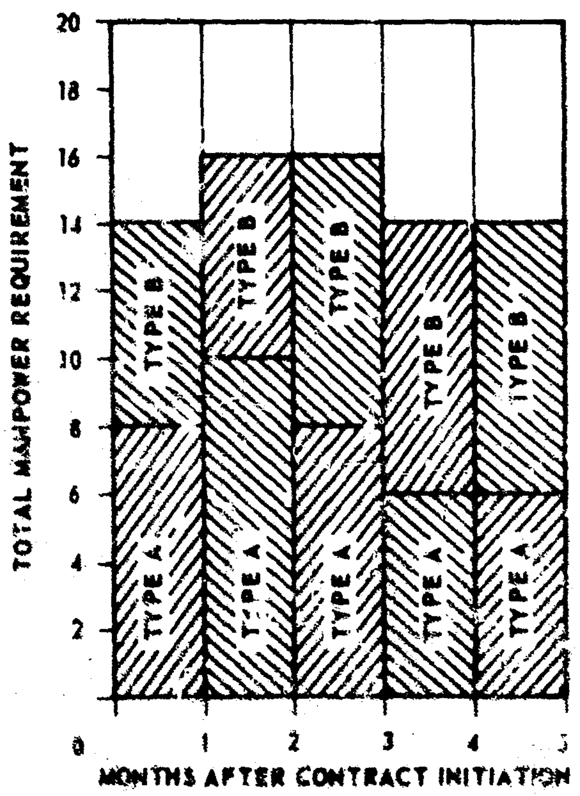

Figure A-1. Time Adjusted Manpower Array for Contract I

Problem Solution

If only the expected values versus time are required, this example problem may be solved by either of two methods; however, if the planner chooses to use some planning criteria other than expected values, the

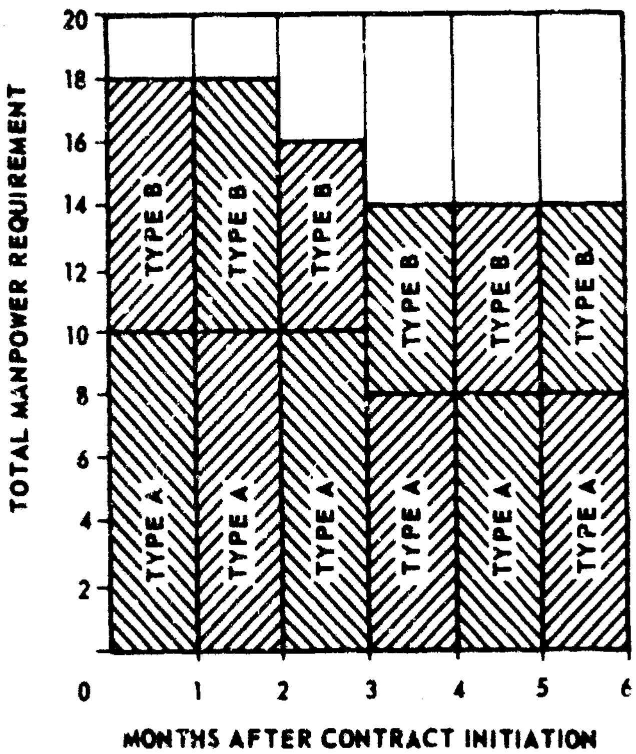

Figure A-2. Time Adjusted Manpower Array for Contract II

enumeration technique should be utilized. Both techniques are demonstrated in the following paragraphs.

Method 1 — Solution by $\left[M \times p_t\right]$ Matrix

As explained in Chapter II, Model III, a $\left[M \times p_t\right]$ matrix is established for each contract proposal and each type of manpower where table entries are calculated by the equation

$$E\left(X_t\right) = p_i\, p_{t/i\,j} M_i\left(t_o + k\right),$$

where

i is the identification of the contract proposal.

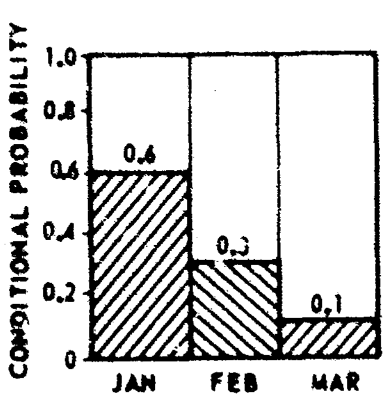

Figure A-3. Probable Initiation Dates for Contract I

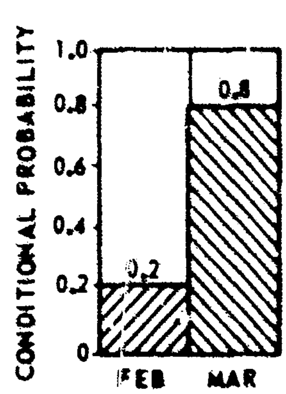

Figure A-4. Probable Initiation Dates for Contract II

k is the time period after contract go-ahead $\left(k = t - t_o\right)$.

t_o is the time period in which it is assumed that contract go-ahead will be received.

t is the time period for which the manpower estimate is being calculated.

p_i is the probability that the i^{th} contract will be received.

$p_{t/i}$ is the probability that contract i, if received, will be initiated during the period t_o.

${}_jM_j\left(t_o + k\right)$ is the number of type j men required for contract i during the period t.

Results of the required calculations are presented in Tables A-1 through A-4.

Table A-1. Expected Manpower Requirement: Type A, Contract I

k	M	t / Mp_1 \ p_t	Jan	Feb	Mar	Apr	May	June	Jul	Aug	Sept
			0.6	0.3	0.1	0	0	0	0	0	0
0-1	8	6.4	3.84	1.92	0.64						
1-2	10	8.0		4.80	2.40	0.80					
2-3	8	6.4			3.84	1.92	0.64				
3-4	6	4.8				2.88	1.44	0.48			
4-5	6	4.8					2.88	1.44	0.48		
$E\left(M_t\right)$			3.84	6.72	6.88	5.60	4.96	1.92	0.48		

Table A-2. Expected Manpower Requirement: Type B, Contract I

k	M	t / Mp_1 \ p_t	Jan	Feb	Mar	Apr	May	June	Jul	Aug	Sept
			0.6	0.3	0.1	0	0	0	0	0	0
0-1	6	4.8	2.88	1.44	0.48						

Table A-2. Expected Manpower Requirement: Type B, Contract I (Concluded)

t			Jan	Feb	Mar	Apr	May	June	Jul	Aug	Sept
k	M	Mp_1 \ p_t	0.6	0.3	0.1	0	0	0	0	0	0
1-2	6	4.8		2.88	1.44	0.48					
2-3	8	6.4			3.84	1.92	0.64				
3-4	8	6.4				3.84	1.92	0.64			
4-5	8	6.4					3.84	1.92	0.64		
$E\left(M_t\right)$			2.88	4.32	5.76	6.24	6.40	2.56	0.64		

Table A-3. Expected Manpower Requirement: Type A, Contract II

t			Jan	Feb	Mar	Apr	May	June	Jul	Aug	Sept
k	M	Mp_2 \ p_t	0	0.2	0.8	0	0	0	0	0	0
0-1	10	7.0		1.40	5.60						
1-2	10	7.0			1.40	5.60					
2-3	10	7.0				1.40	5.60				
3-4	8	5.6					1.12	4.48			
4-5	8	5.6						1.12	4.48		
5-6	8	5.6							1.12	4.48	
$E\left(M_t\right)$				1.40	7.00	7.00	6.72	5.60	5.60	4.48	

Table A-4. Expected Manpower Requirement: Type B, Contract II

t			Jan	Feb	Mar	Apr	May	June	Jul	Aug	Sept
k	M	Mp_2 \ p_t	0	0.2	0.8	0	0	0	0	0	0
0-1	8	5.6		1.12	4.48						

Table A-4. Expected Manpower Requirement: Type B, Contract II (Concluded)

k	M	t / Mp_2 \ p_t	Jan	Feb	Mar	Apr	May	June	Jul	Aug	Sept
		p_t	0	0.2	0.8	0	0	0	0	0	0
1-2	8	5.6			1.12	4.48					
2-3	6	4.2				0.84	3.36				
3-4	6	4.2					0.84	3.36			
4-5	6	4.2						0.84	3.36		
5-6	6	4.2							0.84	3.36	
$E(M_t)$				1.12	5.60	5.32	4.20	4.20	4.20	3.36	

By summing the expected manpower requirements from Tables A-1 through A-4, the total corporate expected manpower requirement was determined to be as shown in Tables A-5 and A-6.

Table A-5. Expected Manpower Requirement: Type A

Contract	Jan	Feb	Mar	Apr	May	June	Jul	Aug	Sept
I	3.84	6.72	6.88	5.60	4.96	1.92	0.48		
II		1.40	7.00	7.00	6.72	5.60	5.60	4.48	
Total	3.84	8.12	13.88	12.60	11.68	7.52	6.08	4.48	

Table A-6. Expected Manpower Requirement: Type B

Contract	Jan	Feb	Mar	Apr	May	June	Jul	Aug	Sept
I	2.88	4.32	5.76	6.24	6.40	2.56	0.64		
II		1.12	5.60	5.32	4.20	4.20	4.20	3.36	
Total	2.88	5.44	11.36	11.56	10.60	6.76	4.84	3.36	

Method 2 — Solution by Complete Enumeration

Utilizing the complete enumeration model for two outstanding proposals (presented in Figure A-5) developed in Chapter II, Model III, the probability of requiring specific numbers of men was calculated and the results are indicated in Tables A-7 and A-8. Combining the probabilities for identical manpower requirements during given months, the probabilistic manpower requirement by months was determined to be as shown in Tables A-9 through A-16, and the total corporate expected manpower requirement versus time was determined to be as shown in Figures A-6 and A-7.

It should be noted that the total corporate expected manpower requirement determined by Methods 1 and 2 are identical.

Assuming straight-line interpolations between the cumulative probability data points, the cumulative distributions for each month were determined to be as shown in Figures A-8 through A-15.

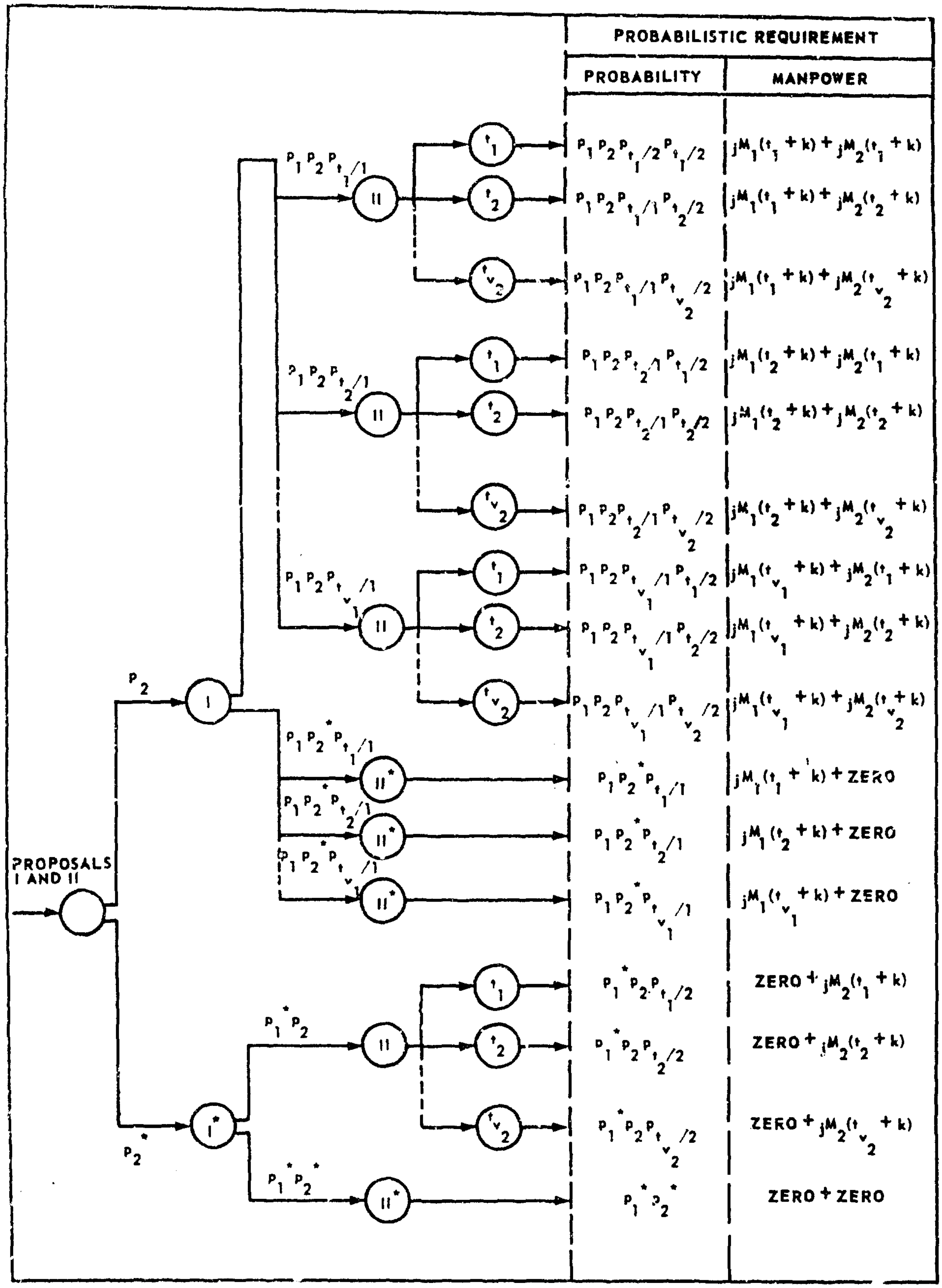

Figure A-5. Enumeration Process for Model III: Two Proposals

Table A-7. Manpower Type A Probabilistic Requirements: Example Problem

Outcome Number	Probability of Outcome	General Equation	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept
1	$p_1p_2p_{t_1}/{}_1p_{t_1}/2$ = (0.8)(0.7)(0.6)(0.2) = 0.0672	${}_jM_1(t_1+k) + {}_jM_2(t_1+k)$	8 + 0 = 8	10 + 10 = 20	8 + 10 = 18	6 + 10 = 16	6 + 8 = 14	0 + 8 = 8	0 + 8 = 8	0 + 0 = 0	0 + 0 = 0
2	$p_1p_2p_{t_1}/{}_1p_{t_2}/2$ = (0.8)(0.7)(0.6)(0.8) = 0.2688	${}_jM_1(t_1+k) + {}_jM_2(t_2+k)$	8 + 0 = 8	10 + 0 = 10	8 + 10 = 18	6 + 10 = 16	6 + 10 = 16	0 + 8 = 8	0 + 8 = 8	0 + 8 = 8	0 + 0 = 0
3	$p_1p_2p_{t_2}/{}_1p_{t_1}/2$ = (0.8)(0.7)(0.3)(0.2) = 0.0336	${}_jM_1(t_2+k) + {}_jM_2(t_1+k)$	0 + 0 = 0	8 + 10 = 18	10 + 10 = 20	8 + 10 = 18	6 + 8 = 14	6 + 8 = 14	0 + 8 = 8	0 + 0 = 0	0 + 0 = 0
4	$p_1p_2p_{t_2}/{}_1p_{t_2}/2$ = (0.8)(0.7)(0.3)(0.8) = 0.1344	${}_jM_1(t_2+k) + {}_jM_2(t_2+k)$	0 + 0 = 0	8 + 0 = 8	10 + 10 = 20	8 + 10 = 18	6 + 10 = 16	6 + 8 = 14	0 + 8 = 8	0 + 8 = 8	0 + 0 = 0
5	$p_1p_2p_{t_3}/{}_1p_{t_1}/2$ = (0.8)(0.7)(0.1)(0.2) = 0.0112	${}_jM_1(t_3+k) + {}_jM_2(t_1+k)$	0 + 0 = 0	0 + 10 = 10	8 + 10 = 18	10 + 10 = 20	8 + 8 = 16	6 + 8 = 14	6 + 8 = 14	0 + 0 = 0	0 + 0 = 0
6	$p_1p_2p_{t_3}/{}_1p_{t_2}/2$ = (0.8)(0.7)(0.1)(0.8) = 0.0448	${}_jM_1(t_3+k) + {}_jM_2(t_2+k)$	0 + 0 = 0	0 + 0 = 0	8 + 10 = 18	10 + 10 = 20	8 + 10 = 18	6 + 8 = 14	6 + 8 = 14	0 + 8 = 8	0 + 0 = 0
7	$p_1p_2'p_{t_1}/1$ = (0.8)(0.3)(0.6) = 0.1440	${}_jM_1(t_1+k)$	8 = 8	10 = 10	8 = 8	6 = 6	6 = 6	0 = 0	0 = 0	0 = 0	0 = 0
8	$p_1p_2'p_{t_2}/1$ = (0.8)(0.3)(0.3) = 0.0720	${}_jM_1(t_2+k)$	0 = 0	8 = 8	10 = 10	8 = 8	6 = 6	6 = 6	0 = 0	0 = 0	0 = 0
9	$p_1p_2'p_{t_3}/1$ = (0.8)(0.3)(0.1) = 0.0240	${}_jM_1(t_3+k)$	0 = 0	0 = 0	8 = 8	10 = 10	8 = 8	6 = 6	6 = 6	0 = 0	0 = 0
10	$p_1'p_2p_{t_1}/2$ = (0.2)(0.7)(0.2) = 0.0280	${}_jM_2(t_1+k)$	0 = 0	10 = 10	10 = 10	10 = 10	8 = 8	8 = 8	8 = 8	0 = 0	0 = 0
11	$p_1'p_2p_{t_2}/2$ = (0.2)(0.7)(0.8) = 0.1120	${}_jM_2(t_2+k)$	0 = 0	0 = 0	10 = 10	10 = 10	10 = 10	8 = 8	8 = 8	8 = 8	0 = 0
12	$p_1'p_2'$ = (0.2)(0.3) = 0.0600	0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0

Table A-8. Manpower Type B Probabilistic Requirements: Example Problem

Outcome Number	Probability of Outcome	General Equation	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept
1	$p_1p_2p_{t_1/1}p_{t_1/2} = (0.8)(0.7)(0.6)(0.2) = 0.0672$	${}_jM_1(t_1+k) + {}_jM_2(t_1+k)$	6 + 0 = 6	6 + 8 = 14	8 + 8 = 16	8 + 6 = 14	8 + 6 = 14	0 + 6 = 6	0 + 6 = 6	0 + 0 = 0	0 + 0 = 0
2	$p_1p_2p_{t_1/1}p_{t_2/2} = (0.8)(0.7)(0.6)(0.8) = 0.2688$	${}_jM_1(t_1+k) + {}_jM_2(t_2+k)$	6 + 0 = 6	6 + 0 = 6	8 + 8 = 16	8 + 8 = 16	8 + 6 = 14	0 + 6 = 6	0 + 6 = 6	0 + 6 = 6	0 + 0 = 0
3	$p_1p_2p_{t_2/1}p_{t_1/2} = (0.8)(0.7)(0.3)(0.2) = 0.0336$	${}_jM_1(t_2+k) + {}_jM_2(t_1+k)$	0 + 0 = 0	6 + 8 = 14	6 + 8 = 14	8 + 6 = 14	8 + 6 = 14	8 + 6 = 14	0 + 6 = 6	0 + 0 = 0	0 + 0 = 0
4	$p_1p_2p_{t_2/1}p_{t_2/2} = (0.8)(0.7)(0.3)(0.8) = 0.1344$	${}_jM_1(t_2+k) + {}_jM_2(t_2+k)$	0 + 0 = 0	6 + 0 = 6	6 + 8 = 14	8 + 8 = 16	8 + 6 = 14	8 + 6 = 14	0 + 6 = 6	0 + 6 = 6	0 + 0 = 0
5	$p_1p_2p_{t_3/1}p_{t_1/2} = (0.8)(0.7)(0.1)(0.2) = 0.0112$	${}_jM_1(t_3+k) + {}_jM_2(t_1+k)$	0 + 0 = 0	0 + 8 = 8	6 + 8 = 14	6 + 6 = 12	8 + 6 = 14	8 + 6 = 14	8 + 6 = 14	0 + 0 = 0	0 + 0 = 0
6	$p_1p_2p_{t_3/1}p_{t_2/2} = (0.8)(0.7)(0.1)(0.8) = 0.0448$	${}_jM_1(t_3+k) + {}_jM_2(t_2+k)$	0 + 0 = 0	0 + 0 = 0	6 + 8 = 14	6 + 8 = 14	8 + 6 = 14	8 + 6 = 14	8 + 6 = 14	0 + 6 = 6	0 + 0 = 0
7	$p_1p_2^*p_{t_1/1} = (0.8)(0.3)(0.6) = 0.1440$	${}_jM_1(t_1+k)$	6 = 6	6 = 6	8 = 8	8 = 8	8 = 8	0 = 0	0 = 0	0 = 0	0 = 0
8	$p_1p_2^*p_{t_2/1} = (0.8)(0.3)(0.3) = 0.0720$	${}_jM_1(t_2+k)$	0 = 0	6 = 6	6 = 6	8 = 8	8 = 8	8 = 8	0 = 0	0 = 0	0 = 0
9	$p_1p_2^*p_{t_3/1} = (0.8)(0.3)(0.1) = 0.0240$	${}_jM_1(t_3+k)$	0 = 0	0 = 0	6 = 6	6 = 6	8 = 8	8 = 8	8 = 8	0 = 0	0 = 0
10	$p_1^*p_2p_{t_1/2} = (0.2)(0.7)(0.2) = 0.0280$	${}_jM_2(t_1+k)$	0 = 0	8 = 3	8 = 8	6 = 6	6 = 6	6 = 6	6 = 6	0 = 0	0 = 0
11	$p_1^*p_2p_{t_2/2} = (0.2)(0.7)(0.8) = 0.1120$	${}_jM_2(t_2+k)$	0 = 0	0 = 0	8 = 8	8 = 8	6 = 6	6 = 6	6 = 6	6 = 6	0 = 0
12	$p_1^*p_2^* = (0.2)(0.3) = 0.0600$	0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0	0 = 0

Table A-9. Probabilistic Manpower Requirements for January: Example Problem

Manpower Type A				Manpower Type B			
Manpower	Probability	Cumulative Probability	Expected Value	Manpower	Probability	Cumulative Probability	Expected Value
0	0.5200	0.5200	0.00	0	0.5200	0.5200	0.00
8	0.4800	1.0000	3.84	6	0.4800	1.0000	2.88
			Σ = 3.84				Σ = 2.88

Table A-10. Probabilistic Manpower Requirements for February: Example Problem

Manpower Type A				Manpower Type B			
Manpower	Probability	Cumulative Probability	Expected Value	Manpower	Probability	Cumulative Probability	Expected Value
0	0.2408	0.2408	0.00	0	0.2408	0.2408	0.00
8	0.2064	0.4472	1.65	6	0.6192	0.8600	3.72
10	0.4520	0.8992	4.52	8	0.0392	0.8992	0.31
18	0.0336	0.9328	0.60	14	0.1008	1.0000	1.41
20	0.0672	1.0000	1.34				Σ = 5.44
			Σ = 8.11				

Table A-11. Probabilistic Manpower Requirements for March: Example Problem

Manpower Type A				Manpower Type B			
Manpower	Probability	Cumulative Probability	Expected Value	Manpower	Probability	Cumulative Probability	Expected Value
0	0.0600	0.0600	0.00	0	0.0600	0.0600	0.00
8	0.1680	0.2280	1.34	6	0.0960	0.1560	0.58
10	0.2120	0.4400	2.12	8	0.2840	0.4400	2.27
18	0.3920	0.8320	7.06	14	0.2240	0.6640	3.14

Table A-11. Probabilistic Manpower Requirements for March: Example Problem (Concluded)

Manpower Type A				Manpower Type B			
Manpower	Probability	Cumulative Probability	Expected Value	Manpower	Probability	Cumulative Probability	Expected Value
20	0.1680	1.0000	3.36	16	0.3360	1.0000	5.38
			Σ = 13.88				Σ: 11.37

Table A-12. Probabilistic Manpower Requirements for April: Example Problem

Manpower Type A				Manpower Type B			
Manpower	Probability	Cumulative Probability	Expected Value	Manpower	Probability	Cumulative Probability	Expected Value
0	0.0600	0.0600	0.00	0	0.0600	0.0600	0.00
6	0.1440	0.2040	0.86	6	0.0520	0.1120	0.31
8	0.0720	0.2760	0.58	8	0.3280	0.4400	2.62
10	0.1640	0.4400	1.64	12	0.0112	0.4512	0.13
16	0.3360	0.7760	5.38	14	0.1456	0.5968	2.04
18	0.1680	0.9440	3.02	16	0.4032	1.0000	6.45
20	0.0560	1.0000	1.12				Σ = 11.55
			Σ = 12.60				

Table A-13. Probabilistic Manpower Requirements for May: Example Problem

Manpower Type A				Manpower Type B			
Manpower	Probability	Cumulative Probability	Expected Value	Manpower	Probability	Cumulative Probability	Expected Value
0	0.0600	0.0600	0.00	0	0.0600	0.0600	0.00
6	0.2160	0.2760	1.30	6	0.1400	0.2000	0.84
8	0.0520	0.3280	0.42	8	0.2400	0.4400	1.92

Table A-13. Probabilistic Manpower Requirements for May: Example Problem (Concluded)

Manpower Type A				Manpower Type B			
Manpower	Probability	Cumulative Probability	Expected Value	Manpower	Probability	Cumulative Probability	Expected Value
10	0.1120	0.4400	1.12	14	0.5600	1.0000	7.84
14	0.1008	0.5408	1.41				Σ = 10.60
16	0.4144	0.9552	6.63				
18	0.0448	1.0000	0.81				
			Σ = 11.69				

Table A-14. Probabilistic Manpower Requirements for June: Example Problem

Manpower Type A				Manpower Type B			
Manpower	Probability	Cumulative Probability	Expected Value	Manpower	Probability	Cumulative Probability	Expected Value
0	0.2040	0.2040	0.00	0	0.2040	0.2040	0.00
6	0.0960	0.3000	0.58	6	0.4760	0.6800	2.86
8	0.4760	0.7760	3.81	8	0.0960	0.7760	0.77
14	0.2240	1.0000	3.14	14	0.2240	1.0000	3.14
			Σ = 7.53				Σ = 6.77

Table A-15. Probabilistic Manpower Requirements for July: Example Problem

Manpower Type A				Manpower Type B			
Manpower	Probability	Cumulative Probability	Expected Value	Manpower	Probability	Cumulative Probability	Expected Value
0	0.2760	0.2760	0.00	0	0.2760	0.2760	0.00
6	0.0240	0.3000	0.14	6	0.6440	0.9200	3.86
8	0.6440	0.9440	5.15	8	0.0240	0.9440	0.19
14	0.0560	1.0000	0.78	14	0.0560	1.0000	0.78
			Σ = 6.07				Σ = 4.83

Table A-16. Probabilistic Manpower Requirements for August: Example Problem

Manpower Type A				Manpower Type B			
Manpower	Probability	Cumulative Probability	Expected Value	Manpower	Probability	Cumulative Probability	Expected Value
0	0.4400	0.4400	0.00	0	0.4400	0.4400	0.00
8	0.5600	1.0000	4.48	6	0.5600	1.0000	3.36
			Σ = 4.48				Σ = 3.36

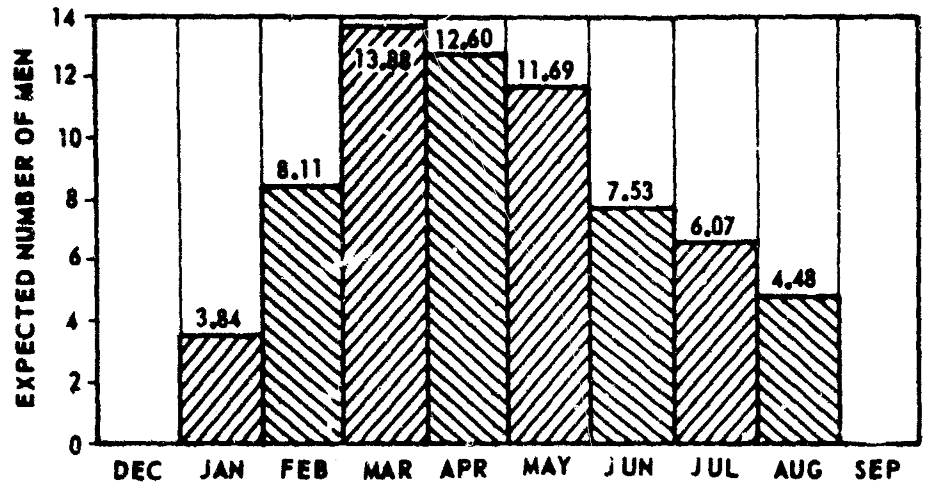

Figure A-6. Corporation Expected Manpower Requirement: Type A

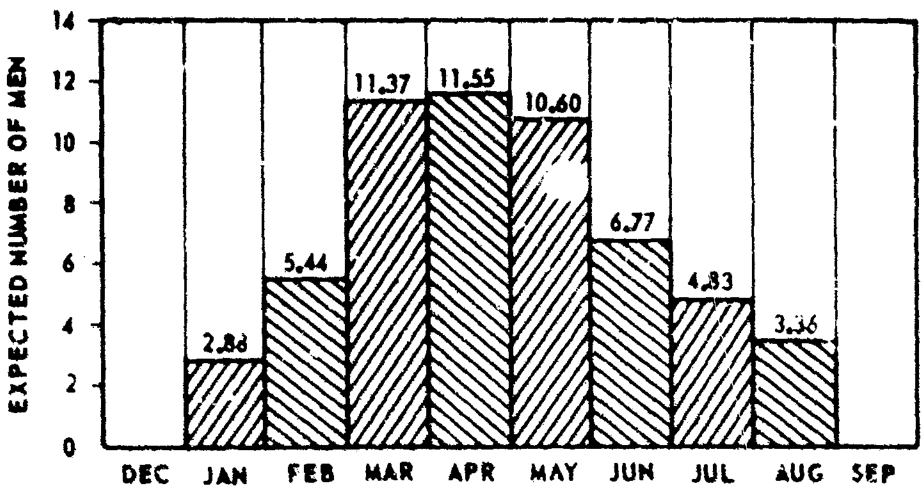

Figure A-7. Corporation Expected Manpower Requirement: Type B

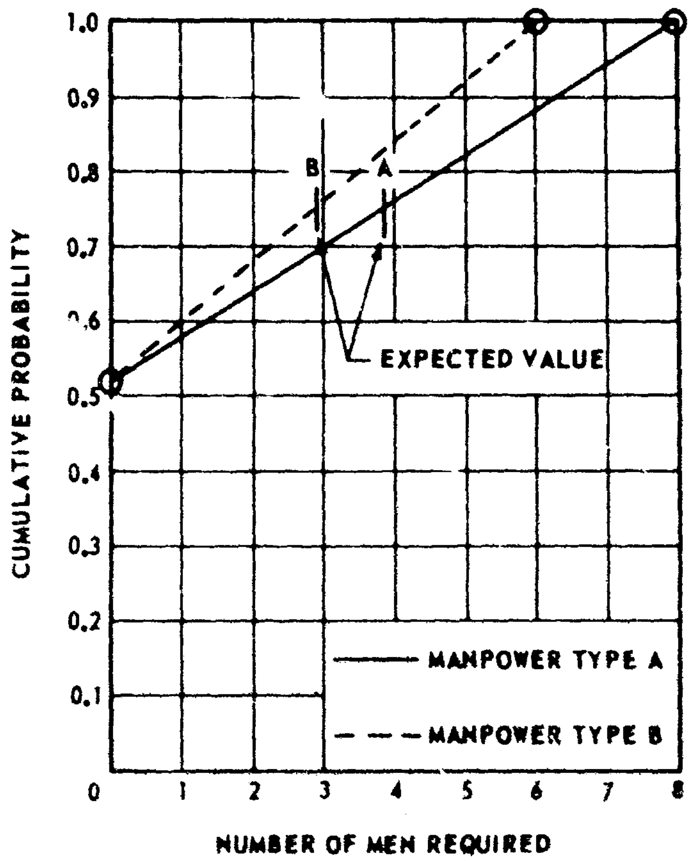

Figure A-8. Probabilistic Manpower Requirements for January

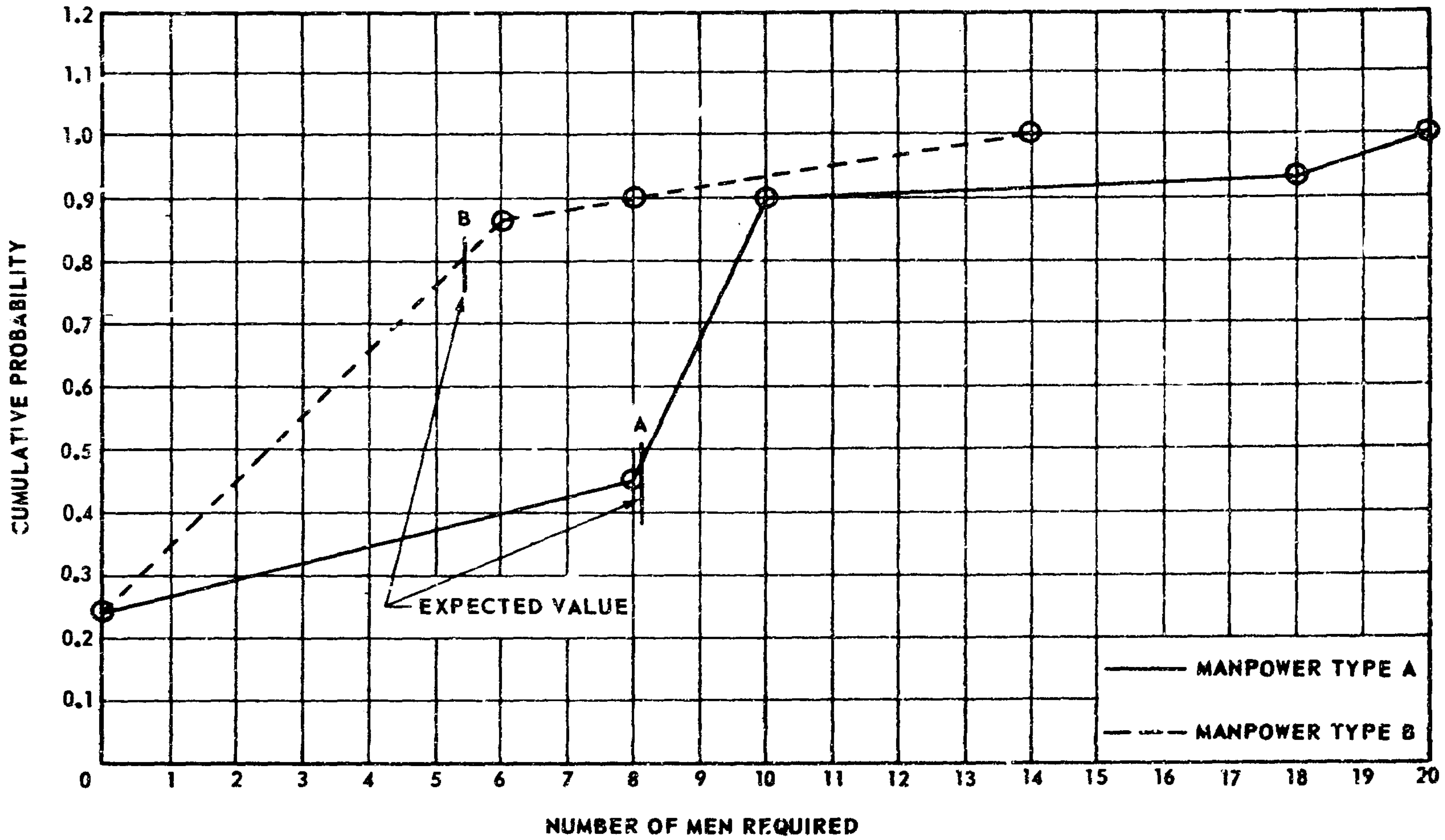

Figure A-9. Probabilistic Manpower Requirements for February

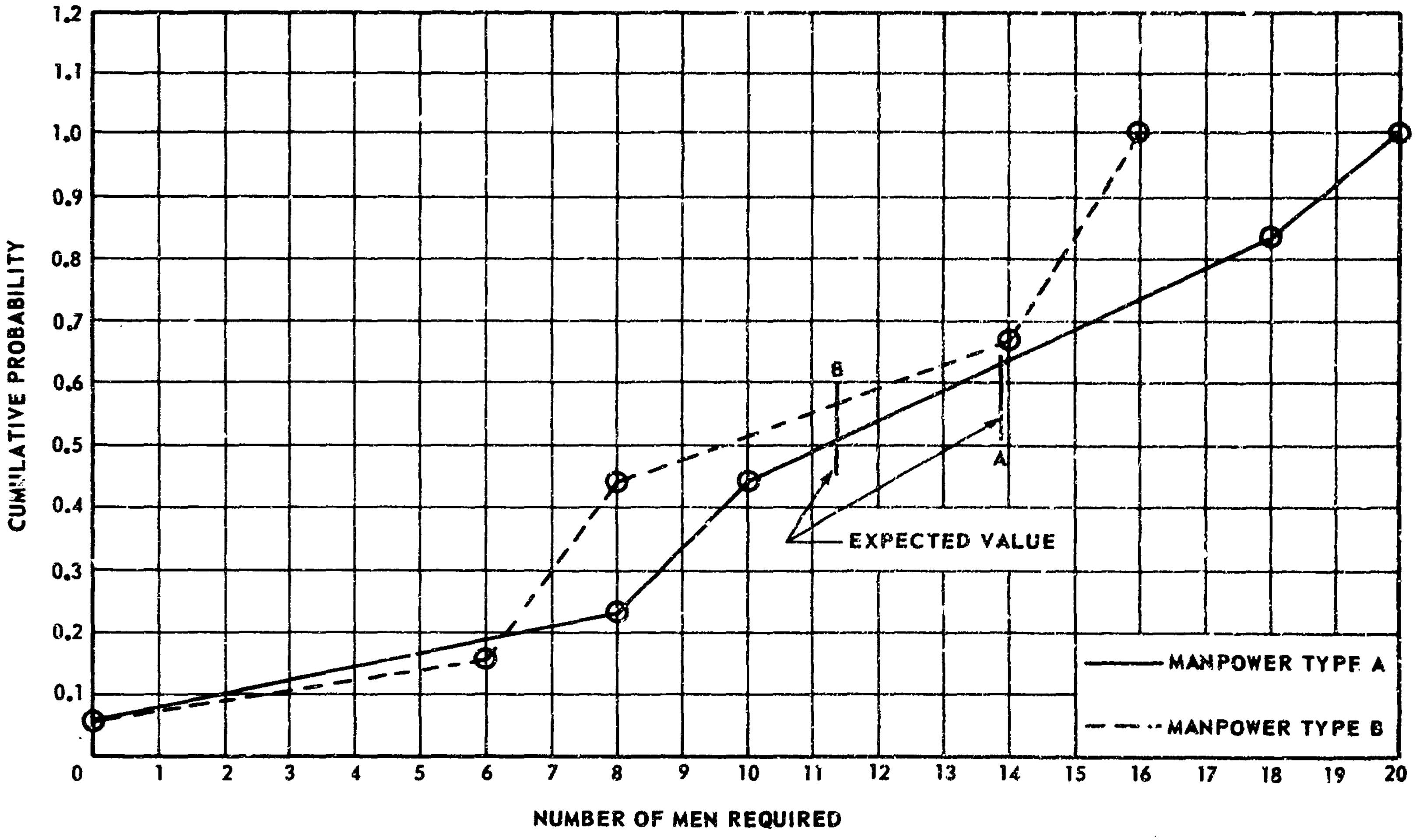

Figure A-10. Probabilistic Manpower Requirements for March

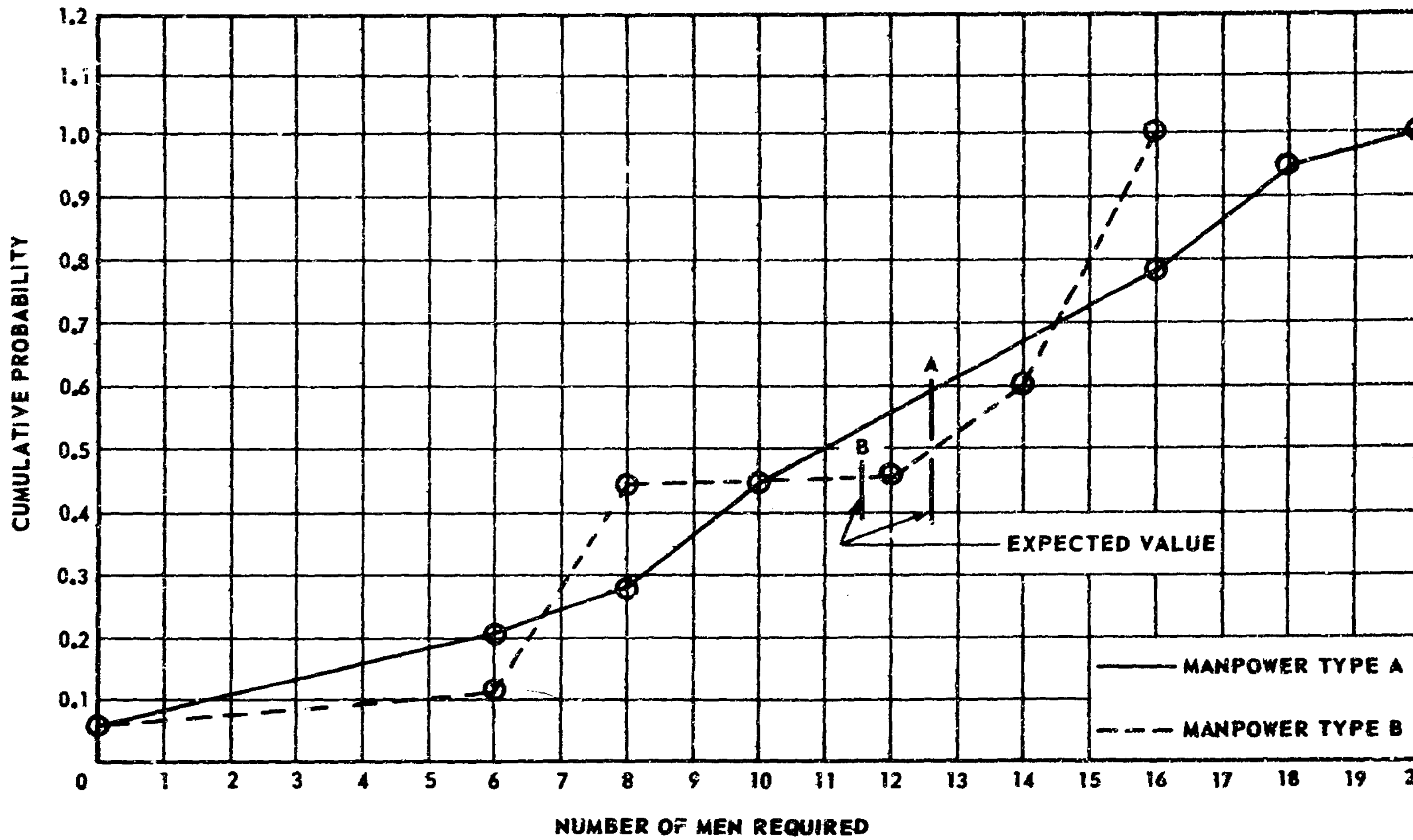

Figure A-11. Probabilistic Manpower Requirements for April

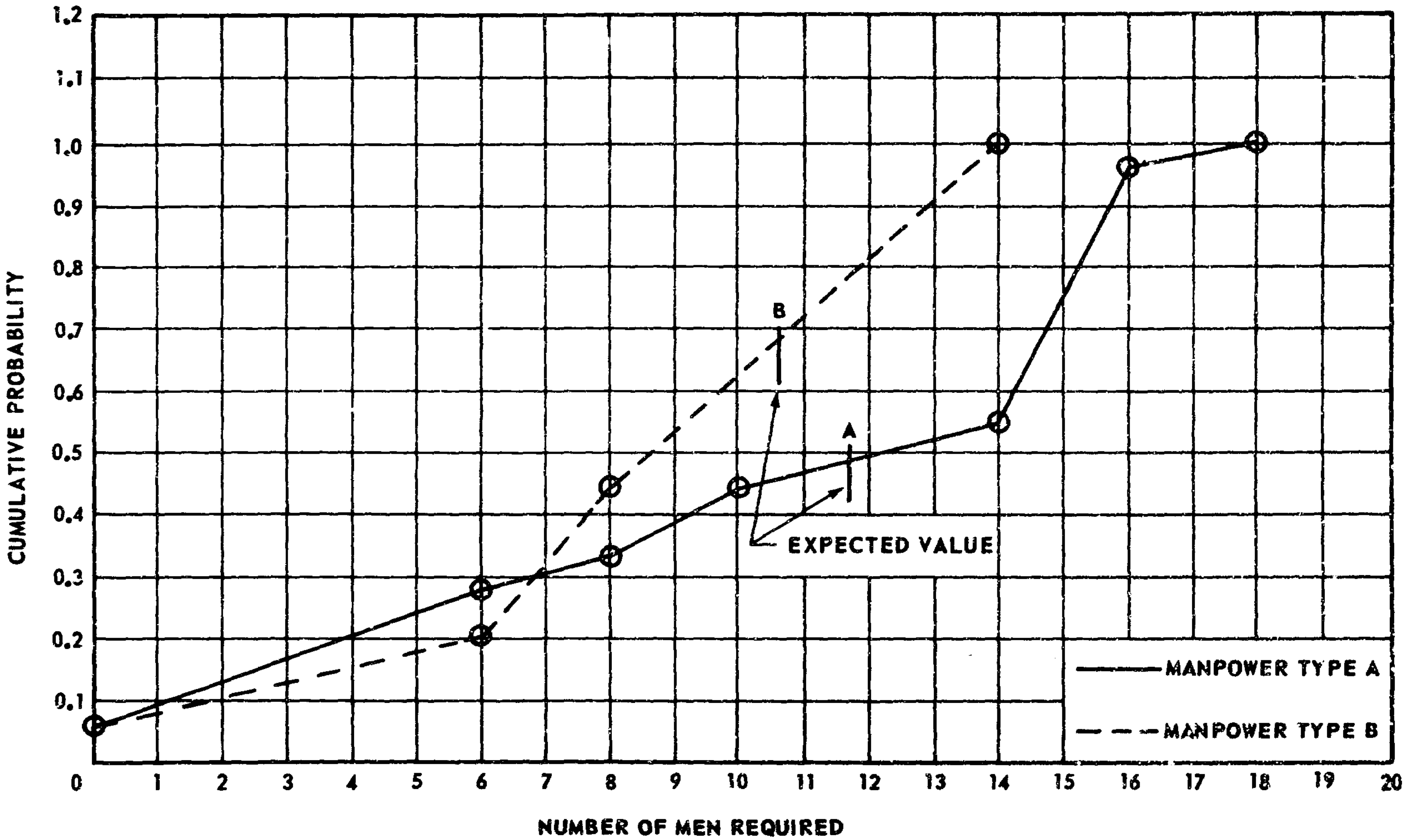

Figure A-12. Probabilistic Manpower Requirements for May

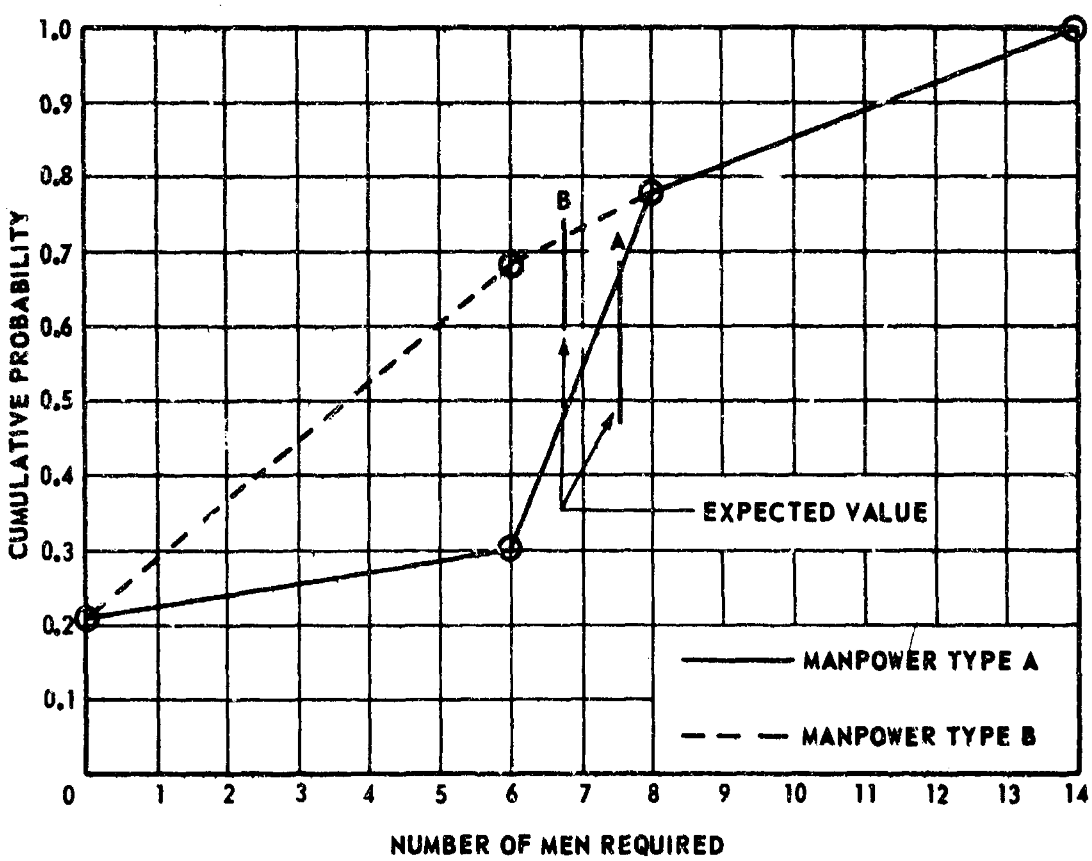

Figure A-13. Probabilistic Manpower Requirements for June

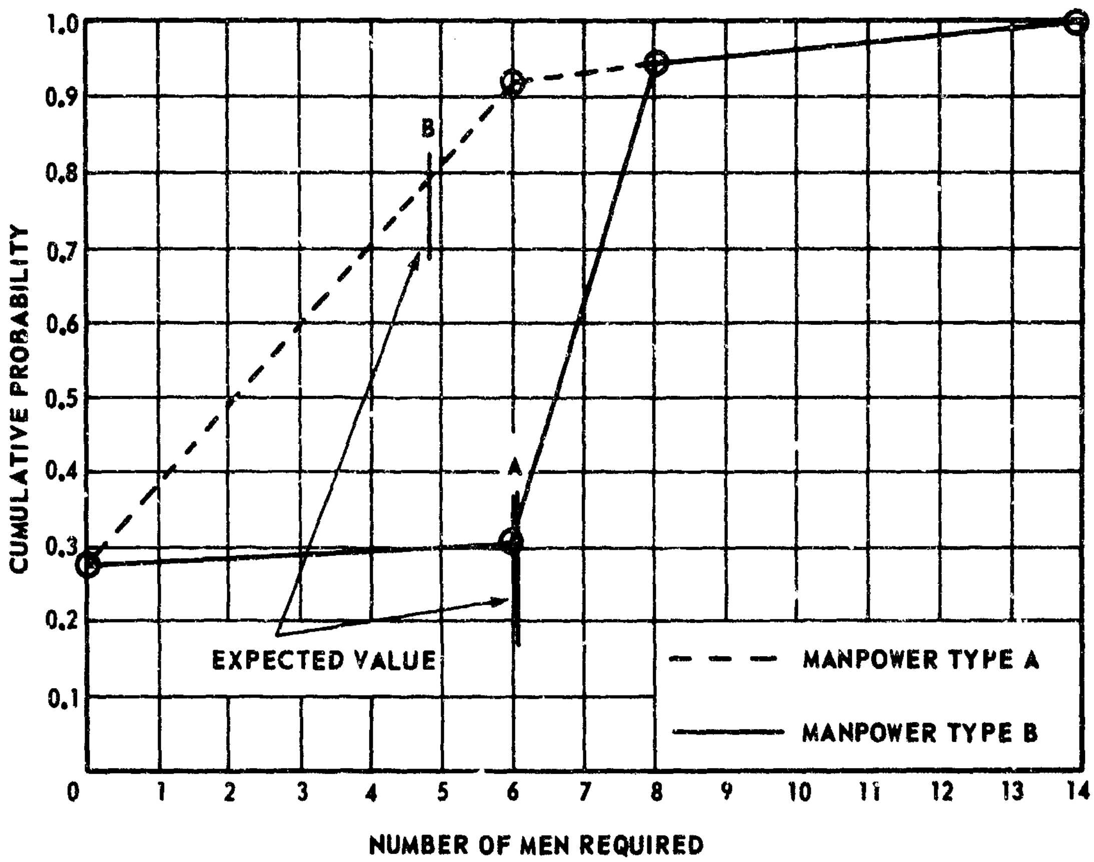

Figure A-14. Probabilistic Manpower Requirements for July

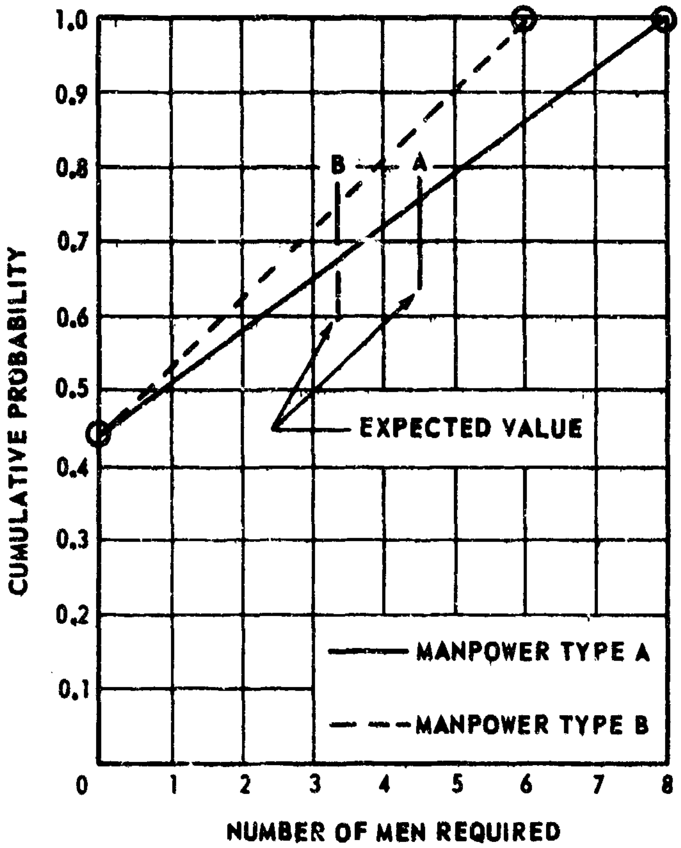

Figure A-15. Probabilistic Manpower Requirements for August

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13. ABSTRACT

There is an existing need for development of techniques for solving inventory problems when the parameters can be specified only in a probabilistic sense. Such a technique was developed in this study which utilized both linear and dynamic programming to operate on the probabilistic data. The problem addressed was manpower for a research and development organization which bids competitively for research and development tasks; however, the technique developed is applicable as well to machines, materials, or any other type of inventory.

Probabilistic planning is visualized as a process for continually and systematically evaluating the manpower requirements and recognizing the risks being taken with any given manpower plan. This author's approach to the manpower problem provides:

1) An array of all possible workloads
2) The probability associated with each
3) The minimum cost approach to performing each workload
4) A corporate manpower plan which allows the corporation to adjust its manpower to the actual workload, when it occurs, with least regret.

Mathematical models developed are suitable for solution on electronic computers and provide management with a rapid evaluation of possible management decisions. The corporation is thus provided with a mathematical simulation of the manpower planning system which can be utilized to readily evaluate effects of various inputs and promote better understanding of the problem at hand.

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Statistical inventory problems						
Linear programming						
Dynamic programming						
Mathematical simulation						